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Inflation Dynamics with a Generalized Lucas Phillips Curve

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I investigate a generalized form of the Lucas (1972, 1973) Phillips curve, that allows different firms to learn aggregate information at different speeds, in otherwise standard textbook new-Keynesian models. This Phillips curve helps to reconcile the sharp divergence between the standard model on the one hand and the beliefs of most policy analysts and the available evidence on the other. In the standard model, inflation and output rise after interest rates rise, with only the possibility of a one-time downward jump. With the generalized Lucas Phillips curve, a small initial disinflation builds up before turning around. With long-term debt, higher interest rates can temporarily lower future inflation with no change in fiscal policy. The model preserves the desirable long-run stability and neutrality of the standard model. The model is tractable, with textbook simplicity and analytic solutions. Lagged inflation in the Phillips curve does not produce a comparable result.

*Hoover Institution, Stanford University and NBER. john.cochrane@stanford.edu, https://www.johnhcochrane.com. I thank six thoughtful referees, Adrian Auclert, and attendees at the Stanford Macro Lunch for helpful comments. Refine.ink made helpful comments and found many errors. A companion paper Cochrane (2026b), Hoover Institution Economics Working Paper 26122, contains much material from an earlier working paper under the title of this article.

1 Introduction

This paper investigates a generalization of the Lucas (1972) Phillips curve, in which some firms take longer than others to figure out whether a price change represents aggregate or relative demand. The output gap x_t follows

$$\kappa x_t = p_t - \int_{j=0}^{\infty} \alpha_j E_{t-j} p_t dj \tag{1}$$

where p_t is the log price level, and α_j is the fraction of firms that take j periods to learn the aggregate state of the economy. I place this Phillips curve in the standard new-Keynesian framework, i.e. an intertemporal IS curve and an interest-rate target. The resulting response of inflation to shocks is hump-shaped. Inflation is initially unstable, building up over time. Later on, inflation dynamics become stable, and inflation goes away.

Specifically, I start by uniting the Phillips curve (1) with the standard intertemporal substitution IS curve,

$$\frac{E_t(dx_t)}{dt} = \sigma(i_t - \pi_t), \tag{2}$$

and I compute the response to a shock consisting of an AR(1) interest rate and an -0.5% initial inflation. Figure 1 presents the resulting response of inflation and output, using exponential weights $\alpha_j = ae^{-aj}$. Disinflation grows following the initial shock, in a hump-shaped pattern, before subsiding. That is the central result.

This is an easy computation. With exponential weights $\alpha_j = ae^{-aj}$, the output and price level responses to a time-0 shock in (1) are related by

$$\kappa x_t = e^{-at} p_t. \tag{3}$$

At time t, output is proportional to the fraction $\int_{j=t}^{\infty} ae^{-aj} dj = e^{-at}$ of firms that have not gotten aggregate news, times the price surprise. The IS curve (2) says that the output, interest rate, and inflation responses are related by

$$\frac{dx_t}{dt} = \sigma(i_t - \pi_t). \tag{4}$$

Take the derivative of (3), eliminate output growth from it and (4), and solve the resulting ordinary differential equation linking $dp/dt = \pi_t$, p_t, and i_t.

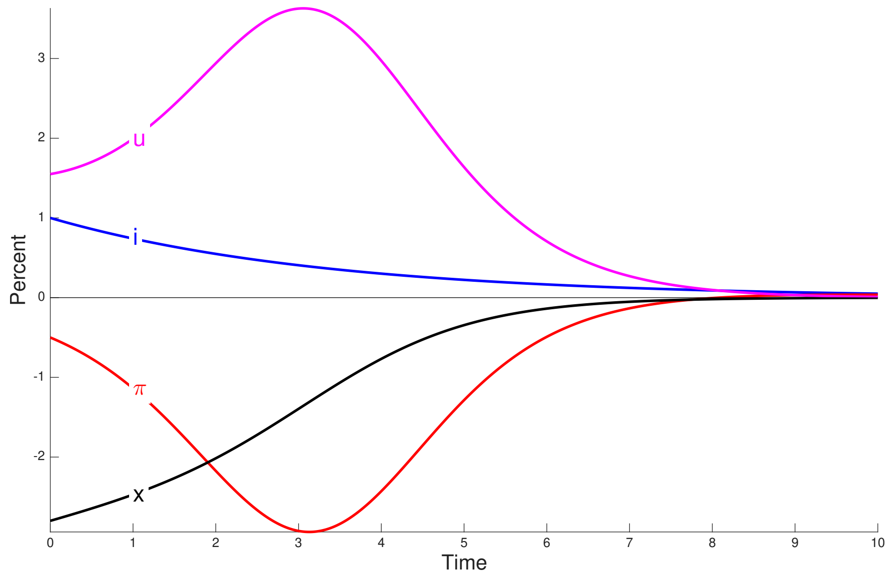

Figure 1: Responses to AR(1) interest rate and $\pi_0 = -0.5\%$ with exponential weights $\alpha_j = ae^{-aj}$. Dynamic IS and a generalized Lucas Phillips curve. Parameters $\sigma = 0.2$, $\kappa = 0.2$, $a = 1$, $b = 0$, $\eta_i = 0.3$. The u line plots u_t in $i_t = \phi\pi_t + u_t$ with $\phi = 1.1$. With $r = 2\%$ and short-term debt, the inflation path requires a fiscal tightening $\tilde{s}/r$ equal to 13.7% of the value of debt.

1.1 Motivation

What is the minimal, simple, baseline, explainable, tractable, textbook-level economic model that captures the sign and dynamics of how nominal interest rates move inflation? Most economists would answer, the standard three-equation new-Keynesian model with a forward-looking Phillips curve. The new-Keynesian literature is well-understood as elaboration of this basic structure.

Yet there is a chasm between the operation of the three-equation model and the standard doctrine among policy makers, commentators, and most economists, taught to undergraduates, and even expressed somewhat incongruously in the introductions, conclusions and executive summaries of many new-Keynesian analyses. In the standard doctrine, higher nominal interest rates raise the real interest rate, which slowly lowers demand, lowers output and employment, and then, more slowly, lowers inflation via the Phillips curve. In the standard three-equation model, by contrast, inflation and output jump down overnight when interest rates rise, and then both *rise* together after that. (See the companion Cochrane, 2026b for a plot and discussion of these standard results.) That rise is deeply embedded in the model's structure: Higher real interest rates raise expected consumption growth by the standard first order condition, and low output lowers inflation relative to future inflation, so inflation rises over time (Ball, 1994). These are the opposite signs of the standard doctrine.

The standard new-Keynesian model retains the Fisherian character of its flexible-price foundation. Higher nominal interest rates, on their own, raise inflation going forward. Price stickiness splits this rise between higher real rates and higher inflation, but does not lower future inflation. Inflation only goes down at all in the presence of higher nominal rates via a coincident equilibrium-selection policy, which induces a fiscal contraction.

Leaving aside the mechanism, there is a similar sharp difference between this model and the facts as understood (tenuously) by VARs, and the standard history of large events such as the 1980s disinflation.

The aim of this paper is to begin to bridge the gap: to work towards a simple, tractable, baseline economic model of inflation under interest-rate targets that captures the signs of the standard doctrine, if not the mechanism of that doctrine.

Many policy makers allude to "long and variable lags" to reconcile the downward jump of simple models and the long decline of the standard doctrine.[1] But modern dynamic economic theory does not allow hand-waving dynamics, or notions of slow adjustment to equilibrium. Each price and quantity must be part of an intertemporal equilibrium. A steady decline of inflation must be a result of a dynamic equilibrium.

That higher interest rates raise inflation fundamentally comes from the stability and long-run neutrality of new-Keynesian models. Given a nominal interest-rate path, inflation will eventually converge to the level of nominal interest rates. By contrast, in the traditional policy view, inflation is unstable given the path of interest rates. Persistently higher nominal interest rates drives inflation down in a spiral. (Cochrane, 2024 explains this stability issue in detail.)

The generalized Lucas Phillips curve bridges that gap. It produces short-run instability while retaining long-run stability. It generates dynamics of the form $d\pi/dt = \gamma(t)\pi_t + \delta(t)i_t$, in which $\gamma(t) > 0$ for small t but $\gamma(t) < 0$ for larger t. In this way, the generalized Lucas Phillips curve produces an inflation response closer to standard beliefs and VARs, but maintaining the long-run stability and neutrality of the rational-expectations model.

However, the mechanism remains fundamentally the same as the standard new-Keynesian model, and differs from that of the policy doctrine. Disinflation still needs an initial downward inflation jump $\pi_0 < 0$ to get going. Higher nominal interest rates, on their own, still just drag

[1] This durable term originates from Milton Friedman (1948), (p. 254). Dupor (2023) has a nice review, and Nelson (2020) (p. 141) discusses the term's coinage.

inflation up. By contrast, the standard doctrine wants higher real interest rates to push inflation down, something like $d\pi/dt = -\gamma(t)(i_t - \pi_t)$ with $\gamma(t) > 0$ for small t, rather than $d\pi/dt = \gamma(t)\pi_t + \delta(t)i_t$.

Mechanically adaptive expectations produce the responses of the standard doctrine (Cochrane, 2024). But the goal of macroeconomic theory since the 1970s has been to avoid such non-economic and hence likely ephemeral and non-policy-invariant ingredients, at least as central necessary ingredients to get the basic sign of monetary policy right.

1.2 Additional Preview

After developing the model and the calculation of Figure 1, I present a case with more realistic hump-shaped weights $\{\alpha_j\}$.

To find inflation, we have to specify an interest-rate path and an equilibrium-selection/fiscal shock, which determines initial inflation and price level decline. That requirement also holds for the model with a standard Calvo Phillips curve, as discussed in the companion paper Cochrane (2026b). I demonstrate the separate effects of interest rates and initial inflation specifications. With short-term debt, both models require an equilibrium-selection shock with a fiscal tightening to get disinflation going. Higher interest rates on their own raise inflation. Thus, there is no reason that those equilibrium-selection shocks should accompany higher interest rates, and no reason why central banks bother to raise interest rates when they want lower inflation.

I then include long-term debt and a persistent interest-rate disturbance. In this case, higher interest rates produce an initial inflation decline despite no change in fiscal policy. This mechanism provides a reason why central banks might want to raise interest rates at all in order to lower inflation. With the standard forward-looking Phillips curve, this long-term debt mechanism still produces a jump down in inflation, followed by a rise. The generalized Lucas Phillips curve amplifies the initial negative inflation, producing more realistic dynamics. With such amplification, one might not notice the initial downward inflation jump in time-averaged data.

The mechanism remains completely different from the standard doctrine, however. The best that anyone has achieved so far is that the rational expectations new-Keynesian model somewhat mimics standard beliefs about the effects of monetary policy — the impulse response function — but not the standard ISLM story for its mechanism.

Mixing the generalized Lucas Phillips curve with a standard intertemporal IS equilibrium condition, output still rises going forward after an initial downward jump, as can be seen in Figure 1. Using an IS curve that features an external habit or one that downweights future output produces response functions in which output also falls before turning around, and output falls more quickly than inflation.

Finally, I show that habit persistence in the IS curve and lagged inflation in the Phillips curve do not on their own produce the hump-shaped inflation response that I pursue.

1.3 Literature Overview

The Appendix includes an extensive literature review. I make a few important points here.

This approach is based on Lucas (1972) and the loglinearization in Lucas (1973). By allowing different firms to learn the aggregate state with greater or lesser delay, I allow persistent output effects. Lucas merged his Phillips curve with a money-demand curve, with money supply determining inflation. I merge the generalized Lucas Phillips curve with an interest-rate target and IS curve, which changes the model dynamics and makes it more relevant to current theory, institutions, and empirical challenges. Moreover, the generalization is more plausible today. While in 1972 it may have seemed reasonable that everyone knew aggregate demand perfectly as soon as money stock numbers were published, that swift information diffusion is less compelling today and with interest-rate targets. Economists are still debating whether the 2021 inflation surge came from aggregate (deficits) vs. relative (supply chain) shocks. Typical businesses may not distinguish relative from aggregate demand even when the national CPI is published, since they operate in local markets.

In more recent literature, the generalized Lucas Phillips curve is closest to the “sticky information” Phillips curve of Mankiw and Reis (2002, 2007). The main technical contribution relative to Mankiw and Reis is that this Phillips curve leads to a simple analytically solvable model, while Mankiw and Reis (2002) only provide an illustrative calculation using a money supply shock, and Mankiw and Reis (2007) require a complex and non-standard numerical solution to incorporate an interest-rate target. (Verona and Wolters, 2013 show how to solve the Mankiw-Reis model in Dynare, by using a large rather than infinite number of state variables.)

Medium-scale numerical models such as Christiano, Eichenbaum, and Trabandt (2016, 2018) and Smets and Wouters (2007) can also produce delayed inflation responses, but with a smorgasbord of additional ingredients.

Less is more. This paper's contribution is not a fundamentally new Phillips curve. Its contribution is to show how small modifications of an "old" Phillips curve can be much more useful than previously known. This paper's contribution is progress towards a simple, minimalist, analytically tractable, textbook model with clear economic intuition that produces a delayed inflation response. Its analytical solutions are even easier than those of the standard new-Keynesian model.

Analytical tractability is important. Hundreds of articles have advanced important theories of price stickiness. But they do not produce an aggregate Phillips curve or other usable summary that can be included in standard DSGE models. As a result, they have not been widely adopted in later work. Other modifications that can be included in numerical models remain black boxes. They are useful and important for quantitative policy evaluation. But without an underlying basic transparent model that can describe the sign and mechanisms of responses, it is hard to say just why they work. Only a tractable model that allows analytic solutions will ever displace the Calvo Phillips curve as the basic blackboard, textbook model on which others build. That is the immodest goal of this article.

Standard new-Keynesian models also rely centrally on an equilibrium-selection/fiscal shock coincident with higher interest rates in order to reduce inflation at all, as explained in the companion Cochrane (2026b). This article builds on that insight, and displays how the generalized Lucas Phillips curve ameliorates the issue by amplifying a small initial shock.

Adaptive expectations and classic IS-LM embody the standard doctrine of monetary policy, and easily produce inflation that goes down following a rise in interest rates. That interest-rate rise must still be accompanied by a fiscal tightening to pay interest costs on the debt, but the mechanism does not rely on an equilibrium-selection shock to get inflation going. However, my goal is to find a fully economic baseline model. If there is no such model, if the basic sign of monetary policy *requires* irrational expectations, that is news. Let us not throw out the entire Lucas-critique-compliant new-Keynesian DSGE enterprise too quickly.

Lagged inflation in the Phillips curve is commonly used to produce a hump-shaped response, justified by price-indexing, rule-of-thumb price-setters, or menu costs with a fixed element. But when combined with the standard IS curve, lagged inflation in the Phillips curve actually makes matters worse, causing inflation to rise more quickly when nominal interest rates rise. Moreover, if that is the central necessary ingredient for the sign of monetary policy, it rests on shaky foundations.

2 A Generalized Lucas Model

Again, I investigate a generalization of the Lucas Phillips curve,

$$\kappa x_t = p_t - \int_{j=0}^{\infty} \alpha_j E_{t-j} p_t dj. \tag{5}$$

One can do all the computations in discrete time, starting with

$$\kappa x_t = p_t - \sum_{j=1}^{\infty} \alpha_j E_{t-j} p_t.$$

Continuous time forces us to think about which variables can jump or diffuse in response to shocks and which must move differentiably. The formulas and pictures are more elegant as well.

One can arrive quickly at (5) starting from the Lucas (1973) supply curve for firm j, in my notation

$$\kappa x_{j,t} = p_{j,t} - E(p_t | I_{j,t})$$

where $I_{j,t}$ is the information set of firm j at time t. The firm expands output in response to its perceived relative price.

Firm j forms its expectations of the aggregate price level at time t based on aggregate information j periods old. To keep the answer simple, I assume that aggregate shocks are much smaller than idiosyncratic shocks. Then, the firm does not update its estimate of the aggregate price level from observations of its own price between time $t - j$ and t. This simplifying assumption could also come from behavioral, information-processing cost, and near-rationality arguments. Given the bewildering number of goods in any advanced economy, it seems reasonable. Averaging over firms, with a fraction α_j firms of type j, we arrive at (5).

The weights $\{\alpha_j\}$ are positive and satisfy $\int_{j=0}^{\infty} \alpha_j dj = 1$. I use two examples for calculations, an exponential

$$\alpha_j = a e^{-aj} \tag{6}$$

and a double exponential,

$$\alpha_j = \frac{e^{-aj} - e^{-bj}}{\frac{1}{a} - \frac{1}{b}}. \tag{7}$$

The single exponential leads to simple calculations and captures the main insights. The double exponential has a hump-shaped pattern with $\alpha_0 = 0$, graphed in Figure 2. In this case, no firms figure out instantly whether a disturbance is relative or aggregate. It adds a few simplifications

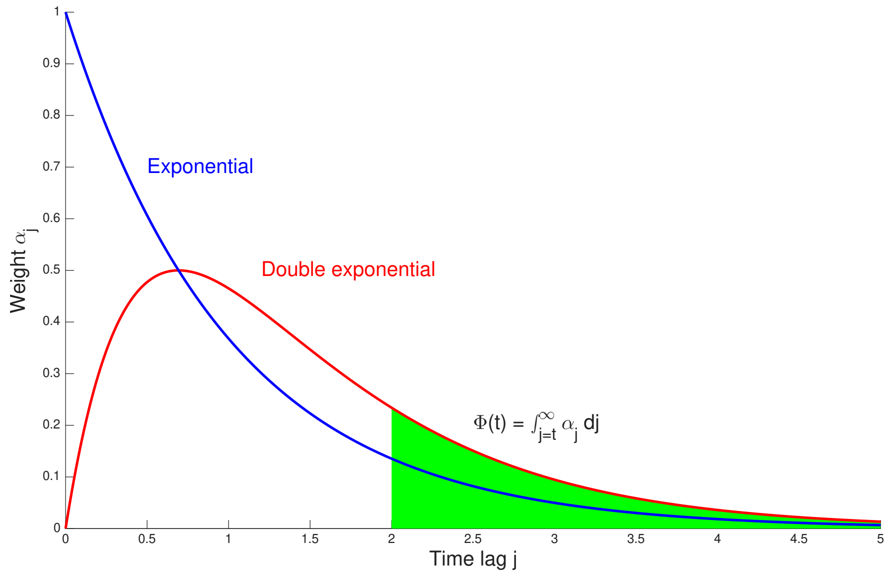

Figure 2: Weights in the generalized Lucas Phillips curve. The plot shows weights α_j for an exponential $\alpha_j = ae^{-aj}$ and double exponential $\alpha_j = (e^{-aj} - e^{-bj})/(1/a - 1/b)$ examples. $a = 1$ and $b = 2$. The shaded area illustrates $\Phi_t \equiv \int_{j=t}^{\infty} \alpha_j dj$ for $t = 2$ in the double-exponential case.

and refinements to the results. It is also a more reasonable extension of Lucas's ideas to continuous time. The peak of the double exponential stands in well for Lucas's "period," with some firms getting news sooner and others later.

I characterize model solutions by impulse-response functions. Since the model is linear, the response function—how $(E_{t+1} - E_t)y_{t+j}$ changes when there is a shock at time $t + 1$, for a variable y_t—is the same as the response to an "MIT shock" at time 0, in which all variables and expectations start at zero and then jump unexpectedly to new values. I use unadorned time-series notation y_t to denote such impulse-response functions.

From the Phillips curve, we can compute how the price level or inflation response to any shock are related to the output response. We have $E_j p_t = 0$ for any $j < 0$, while $E_j p_t = p_t$ for any $j \geq 0$. Thus, the price and output responses are related by

$$\kappa x_t = p_t - \int_{j=0}^{t} \alpha_j dj \; p_t$$

$$\kappa x_t = \left(\int_{j=t}^{\infty} \alpha_j dj \right) \; p_t = \Phi(t) p_t. \tag{8}$$

Output moves in proportion to the fraction of firms that have not yet learned aggregate information. Equation (8) also gives us the output response once we solve for the price level response below.

The last equality in (8) introduces notation $\Phi(t)$ for the fraction of firms that have not yet learned aggregate demand. It is also the right tail of $\{\alpha_j\}$. This fraction obeys

$$\Phi(0) = 1;\ \Phi(\infty) = 0;\ \frac{d\Phi(t)}{dt} = -\alpha_t.$$

For exponential weights (6),

$$\Phi(t) = e^{-at},$$

and for double-exponential weights (7),

$$\Phi(t) = \frac{\frac{1}{a}e^{-at} - \frac{1}{b}e^{-bt}}{\frac{1}{a} - \frac{1}{b}}.$$

We will shortly need the response of output growth. Taking the derivative of (8),

$$\kappa \frac{dx_t}{dt} = -\alpha_t p_t + \Phi(t)\frac{dp_t}{dt}. \tag{9}$$

One can derive a Phillips curve linking output and inflation, in the standard tradition, rather than output and the price level. However, it turns out to be algebraically simpler to solve the model in terms of the price level and then take a derivative.

Only a fraction λdt of firms can change prices between time t and $t + dt$ in the continuous-time version of the standard Calvo (1983) pricing model, so the price level cannot have a jump or diffusion, though the rate of inflation can do so. Prices and output can jump or diffuse with this generalized Lucas Phillips curve. If that result seems counterintuitive, remember that individual firm prices and output flows do jump, and that all our statistics are time-averaged.

3 Model Response With a Dynamic IS curve

How the economy behaves depends on the full model, not just the Phillips curve. The response of the standard intertemporal IS curve to a time-zero shock is, for $t \geq 0$,

$$\frac{dx_t}{dt} = \sigma(i_t - \pi_t). \tag{10}$$

This is the continuous-time equivalent to the familiar

$$x_t = x_{t+1} - \sigma(i_t - \pi_{t+1}).$$

Figure 1 plots the response of the model consisting of the generalized Lucas Phillips curve and the IS curve (10) to the combination of an AR(1) interest-rate path and a $\pi_0 = -0.5\%$ inflation shock.

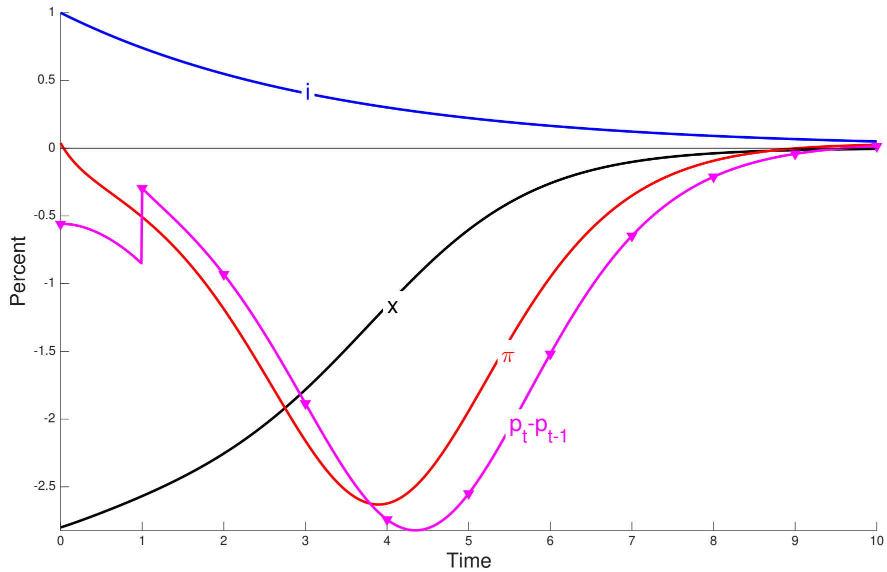

Figure 3: Responses to an AR(1) interest rate and $p_0 = -0.56\%$ with double-exponential weights. Dynamic IS and a generalized Lucas Phillips curve. Parameters $\sigma = 0.2$, $\kappa = 0.2$, $a = 1$, $b = 2$, $\eta_i = 0.3$.

Figure 3 presents the response of inflation and output to the same shock, using a hump-shaped pattern of weights generated by the difference between two exponentials. (See (7) and Figure 2.) The major difference is that inflation does not jump downwards on impact. Avoiding that feature is a main motivation (aside realism) for examining hump-shaped weights. The price level still jumps down, so measured year-on-year inflation declines as shown.

To compute these responses, I eliminate output growth from (9) and (10). Rearranging, the price level then follows the differential equation

$$[\Phi(t) + \sigma\kappa]\ \frac{dp_t}{dt} - \alpha_t p_t = \sigma\kappa i_t. \tag{11}$$

All of the derivatives are forward-looking and so interpreted hold at time zero as well.

Since $\Phi'(t) = -\alpha_t$, the left-hand side is the total differential of $[\Phi(t) + \sigma\kappa] p_t$. We quickly have the solution

$$p_t = \frac{1+\sigma\kappa}{\Phi(t)+\sigma\kappa} p_0 + \frac{\sigma\kappa}{\Phi(t)+\sigma\kappa} \int_{j=0}^{t} i_j dj. \tag{12}$$

For exponential weights,

$$p_t = \frac{1+\sigma\kappa}{e^{-at}+\sigma\kappa} p_0 + \frac{\sigma\kappa}{e^{-at}+\sigma\kappa} \int_{j=0}^{t} i_j dj. \tag{13}$$

The coefficient in front of p_0 is the central story. The terms $\Phi(t)$ and e^{-at} start at 1 and then decrease. The price level grows or declines over time from an initial p_0, but then stabilizes at $(1+\sigma\kappa)/(\sigma\kappa)$ of that original value. Also in the differential equation (11), the coefficient of dp_t/dt on p_t is positive, so the model exhibits explosive dynamics, amplifying an initial negative shock over time. However, $\alpha_t \to 0$ and $\Phi(t) \to 0$ as t grows, so the price level eventually converges.

I use an AR(1) process for the interest rate

$$i_t = i_0 e^{-\eta_i t}$$

so

$$\int_{j=0}^{t} i_j dj = \frac{1-e^{-\eta_i t}}{\eta_i}.$$

This completes the calculation of the price level path.

To find the inflation response, we can differentiate the price response, yielding

$$\pi_t = \frac{(1+\sigma\kappa)\,\alpha_t}{[\Phi(t)+\sigma\kappa]^2} p_0 + \frac{\sigma\kappa\alpha_t}{[\Phi(t)+\sigma\kappa]^2} \int_{j=0}^{t} i_j dj + \frac{\sigma\kappa}{\Phi(t)+\sigma\kappa} i_t. \tag{14}$$

In the single-exponential case,

$$\pi_t = \frac{(1+\sigma\kappa)\,ae^{-at}}{(e^{-at}+\sigma\kappa)^2} p_0 + \frac{\sigma\kappa ae^{-at}}{(e^{-at}+\sigma\kappa)^2} \int_{j=0}^{t} i_j dj + \frac{\sigma\kappa}{e^{-at}+\sigma\kappa} i_t. \tag{15}$$

Equations (12) and (14) show that we must characterize the policy input by both an interest-rate path and an initial price level, which solves the mystery of why I did so. Given a path of interest rates, there are multiple equilibria, indexed by p_0. Each choice of p_0 carries different fiscal implications. Thus, we can call the choice of p_0 the equilibrium-selection/fiscal policy specification. Holding constant the initial price level p_0, higher nominal interest rates *raise* inflation.

Thus, any disinflationary response must come from amplifying a negative initial price level or negative inflation. For this reason, I plot responses to an AR(1) interest-rate path *and* an initial $\pi_0 = -0.5\%$ or $p_0 = -0.56\%$ price decline.

Since $p_0 < 0$ does all the disinflationary work, and given p_0 raising interest rates only raises inflation, it seems almost dishonest to plot the response to such a joint policy and call it the result of monetary policy. I do so reflecting a long tradition. The same points hold with the standard model with a forward-looking Phillips curve, so this is an analogue of the standard calculation, not something new and different. See the companion Cochrane (2026b). The next section considers other policy specifications.

To understand the behavior at time zero and how it differs across the single- and double-exponential forms, consider the initial conditions. We can read these directly off the differential equation (11) or the inflation response (14) at $t = 0$:

$$(1 + \sigma\kappa)\,\pi_0 - \alpha_0 p_0 = \sigma\kappa i_0. \tag{16}$$

When $\alpha_0 > 0$ as it is for the exponential weights, then the initial price level and inflation rate are tied together. I specify $\pi_0 = -0.5\%$ and find the implied $p_0 = -0.56\%$. One can in this case also solve (16) as

$$p_0 = \frac{1}{\alpha_0}\left[(1 + \sigma\kappa)\,\pi_0 - \sigma\kappa i_0\right], \tag{17}$$

and express inflation in (15) as a function of an initial inflation choice π_0 rather than the initial price level p_0.

When $\alpha_0 = 0$, as in the double exponential weights, initial inflation has no connection to the initial price level. As α_t rises however, the initial price level soon feeds back to the inflation path. Initial inflation must be positive when the initial interest rate is positive, a brief "price puzzle" that one can barely see in Figure 3. I specify $p_0 = -0.56\%$, so the calculation is comparable to the exponential weight case.

The lines marked inflation π in Figures 1 and 3 plot the instantaneous inflation rate, ignoring the price level jump that gets the whole thing going. To display that phenomenon, Figure 3 also plots inflation measured as change from the previous period $p_t - p_{t-1}$. Here we see inflation starting at -0.56%. This value is the initial price level jump less the price level (0) one period earlier. Measured inflation jumps up as the initial price level drop enters the moving average. That seems unrealistic. One might add small frictions to smooth out the model. However, the triangles plot year-on-year inflation at discrete time intervals, which is what we would typically measure. The triangles are not obviously counterfactual. In reality, the price level is also time-averaged to monthly or quarterly intervals, further smoothing the response.

While the double-exponential case is more realistic, I plot the simple exponential response to emphasize that the hump-shaped response does not come from hump-shaped weights. I also focus on the single-exponential case in order to focus on the simplest model that expresses the central ideas.

The central story is initially unstable inflation that becomes stable as time increases. To see this behavior, we can write the differential equation for inflation by differentiating the price differential equation (11), and then using it again to eliminate the price level in favor of inflation. For exponential weights, the result is

$$\frac{d\pi_t}{dt} = \frac{(e^{-at} - \sigma\kappa)}{(e^{-at} + \sigma\kappa)} a\pi_t + \frac{1}{e^{-at} + \sigma\kappa}\left[a\sigma\kappa i_t + \sigma\kappa\frac{di_t}{dt}\right]. \tag{18}$$

For $\sigma\kappa < 1$, inflation is initially unstable, with a positive coefficient on π_t. As t rises and e^{-at} falls, the sign changes and inflation becomes stable in the long run, with a negative coefficient on π_t. For this reason, I calculate the response in Figure 1 with σ and κ relatively small, at 0.2 each. A smaller value of σ and κ allows a generous period of instability before e^{-at} falls below $\sigma\kappa$ and inflation becomes stable. The parameter $a = 1$ means the typical firm learns the aggregate price level in about a year. Smaller values of a, capturing slower learning, slow down the dynamics.

Output x_t follows from the price level (12) and equation (8),

$$\kappa x_t = \left(\int_{j=t}^{\infty} \alpha_j dj\right)\ p_t = \Phi(t)p_t. \tag{19}$$

Whether output rises or falls after an initial jump $\kappa x_0 = p_0$ depends on whether the reduction in $\Phi(t)$ outweighs the growth in p_t. Using (12)

$$\kappa x_t = \frac{\Phi(t) + \Phi(t)\sigma\kappa}{\Phi(t) + \sigma\kappa}\kappa x_0 + \frac{\Phi(t)\sigma\kappa}{\Phi(t) + \sigma\kappa}\int_{j=0}^{t} i_j dj. \tag{20}$$

We see that for $p_0 < 0$ and $x_0 < 0$, the leading coefficient is less than one, and output always reverts to trend, as shown in Figures 1 and 3. Fiddling with parameters will not help. If we want to modify this behavior, we have to modify the model, which I do below.

We can see the basic economic mechanism for inflation dynamics with a static IS curve and the Lucas Phillips curve,

$$x_t = -\sigma\left(i_t - E_t\pi_{t+1}\right)$$

$$\kappa x_t = \pi_t - E_{t-1}\pi_t.$$

Eliminating x_t, we obtain the equilibrium condition

$$E_t\pi_{t+1} = i_t + \frac{1}{\sigma\kappa}(\pi_t - E_{t-1}\pi_t).$$

Denote the impulse response function $\pi_0,\ \pi_1$, etc. following a shock at time 0. Given an interest-rate path $\{i_t\}$ and first-period inflation π_0, inflation follows

$$\pi_0;\ \pi_1 = i_0 + \frac{1}{\sigma\kappa}\pi_0;\ \pi_2 = i_1, , ...\pi_t = i_{t-1}.$$

If $\sigma\kappa < 1$, the period-1 response can amplify the period-0 response. We have initially unstable dynamics that then turn around and become stable and neutral.

This simple case helps shows the intuition: If $\pi_0 > 0$, then we have a positive output response x_0 via the Lucas Phillips curve. In this model, higher output x_0 requires a lower real interest rate. So $i_0 - E_0\pi_1$ must fall. With a fixed nominal rate i_0, that fall requires a rise in π_1 relative to i_0. The rise in π_1 is, when time 1 rolls around, a rise in expected inflation — $\pi_1 = E_0\pi_1$ in the impulse response function — so has no further output effect. Lucas (1972) is famously a model without persistence. But that dictum is restricted to output. Even Lucas' Phillips curve generates inflation persistence when married to an interest-rate target and an IS condition.

4 Policy Specification

It is unconventional that I specify the policy environment by an interest-rate path and also by the initial price level, reflecting an equilibrium-selection/fiscal choice. It also seems unusual that equilibrium-selection choice does all the work of lowering inflation, and that higher nominal interest rates on their own raise inflation.

The issue is not a special problem of the generalized Lucas Phillips curve. As explained in Cochrane (2026b), the issues are the same for the standard forward-looking Calvo Phillips curve.

In that model, the relation between interest rates and inflation is given by

$$\pi_t = \frac{1}{\frac{1}{\lambda_1} + \frac{1}{\lambda_2}} \left[Ce^{-\lambda_1 t} + \int_{s=0}^{t} e^{-\lambda_1(t-s)} i_s ds + \int_{s=t}^{\infty} e^{-\lambda_2(s-t)} i_s ds \right] \tag{21}$$

where λ_1 and λ_2 are positive coefficients that depend on the model parameters. Broadly, (21) works in the same way as (14) and (15). Inflation is a two-sided moving average of the interest rate, plus an exponentially decaying transient which can be indexed by initial inflation π_0. Each choice corresponds to a different fiscal policy to pay interest costs on the debt and windfall or devaluation of nominal bonds. Inflation is determined by an equilibrium-selection/fiscal choice to determine C or π_0 along with the interest-rate path. The weights of inflation on the interest rate are all positive, so for a given equilibrium-selection choice, higher interest rates raise inflation. The only way for inflation to decline with higher interest rates is via a contemporaneous negative equilibrium-selection/fiscal π_0 shock.

There are some differences. Inflation in (21) responds to future interest rates, while only past interest rates enter in the model with a generalized Lucas Phillips curve (14) and (15). The response to the initial inflation shock in (21) is a pure exponential decay, which is why the Calvo model produces a rise in inflation after an initial downward jump. The initially unstable, then stable, dynamics of the generalized Lucas model are by contrast its central innovation.

As explained in detail in Cochrane (2026b), we can understand equilibrium choice in three ways. The most classic is via a Taylor rule

$$i_t = \phi \pi_t + u_t.$$

This specification seems to avoid the equilibrium-selection choice, but it does not. There are many different $\{u_t\}$ that produce a given $\{i_t\}$, and each produces a different initial inflation π_0. We can also write the interest-rate policy rule in the equivalent representation suggested by King (2000),

$$i_t = i_t^* + \phi(\pi_t - \pi_t^*). \tag{22}$$

Here i_t^* is the central bank's desired equilibrium interest-rate path, and π_t^* is the equilibrium the central bank wishes to choose, among the set consistent with $\{i_t^*\}$. This expression makes clear how the central bank chooses an equilibrium π_t^*.

The linearized government debt identity implies

$$\int_{j=0}^{\infty} e^{-rj}\tilde{s}_j dj = \int_{j=0}^{\infty} e^{-rj}(i_j - \pi_j)dj - p_0 + q_0. \tag{23}$$

where $\tilde{s}$ is the real primary surplus divided by the initial (steady state) value of debt and q_0 is the jump in the nominal value of debt when there is long-term debt (Cochrane, 2023). Primary surpluses must pay interest costs on the debt and the windfall gains and losses to bondholders coming from a price-level or bond-price jump.

In the Taylor or King rule vision, surpluses react "passively" to central bank equilibrium-selection choices. In the fiscal theory of the price level, the change in surpluses directly selects the equilibrium. But in either vision, surpluses must satisfy (23) and we can index the various equilibria by their fiscal underpinnings. Inflation is always determined jointly by monetary and fiscal policy.

To illustrate these foundations, I compute the fiscal underpinnings or determinants of each response, the left-hand side of (23). The timing of surpluses does not matter in this model, so I calculate the discounted sum of a permanent change in surplus $\tilde{s}_0/r$. We can compare that quantity to the total outstanding debt. With short-term debt ($q_0 = 0$), this displayed solution requires a fiscal tightening equal to 13.7% of the value of debt. You can see the long period of interest rates larger than inflation, generating interest costs.

Figure 1 displays these three interpretations of the policy specification. First, we can directly view that equilibrium as an instance of the King-rule equilibrium selection policy plus passive fiscal tightening. Second, we can view it an instance of the fiscal theory of the price level, with short-term debt and a 13.7% fiscal tightening contemporaneous with the interest-rate rise. Third, I compute and plot a Taylor rule disturbance $u_t = i_t - \phi\pi_t$ with $\phi = 1.1$. This Taylor rule disturbance produces the same equilibrium.

You might object that I seem to bake in the hump-shaped outcome with a hump-shaped disturbance. But hump-shaped inflation with an AR(1) interest rate and $i_t = \phi\pi_t + u_t$ must correspond to a hump-shaped disturbance. Putting that hump-shaped disturbance in the Calvo model would produce a hump-shaped interest rate, not hump-shaped inflation. VARs typically find something like an AR(1) interest-rate path. They do not measure the Taylor rule disturbance.

So, we can describe the policy experiment of Figure 1 as a response to the plotted Taylor rule disturbance $\{u_t\}$ supported by a "passive" 13.7% fiscal contraction; we can describe it as a response to the plotted interest path and a $\pi_0 = -0.5\%$ King-rule equilibrium-selection shock, again with a "passive" 13.7% fiscal contraction; or we can describe it as the plotted interest-rate path plus an "active" 13.7% fiscal contraction which directly selects initial inflation. Obviously, I find the latter interpretation most compelling, but the reader may choose any interpretation.

4.1 Interest Rates vs. Equilibrium Selection

To make vivid the effects of interest rates vs. equilibrium selection, I examine how the initial inflation or price level assumption vs. the interest-rate path assumption affect inflation.

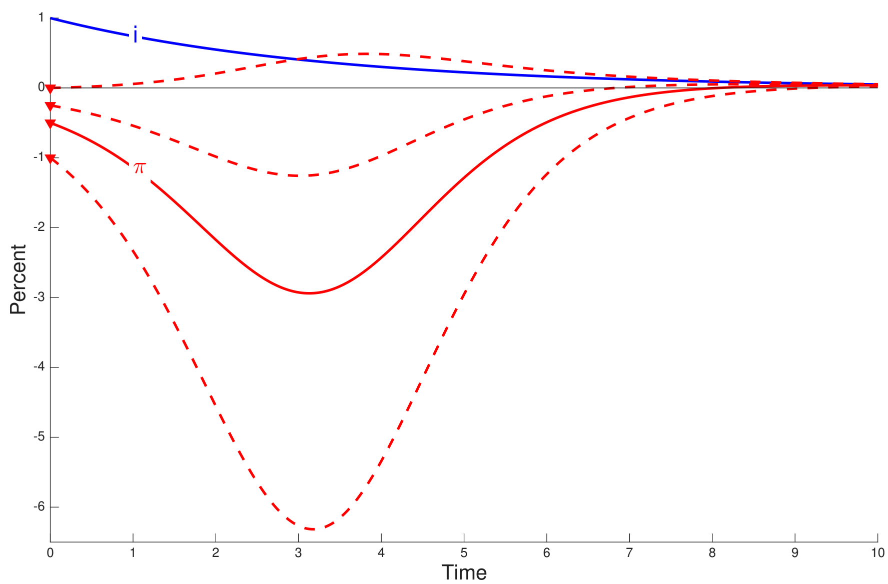

Figure 4: Inflation responses for various values of initial inflation, and the same interest rate. Continuous-time model with a dynamic IS curve and a generalized Lucas Phillips curve. Parameters $\sigma = 0.2$, $\kappa = 0.2$, $a = 1$, $\eta_i = 0.3$. The solid line uses the original value $\pi_0 = -0.5\%$. Dashed lines give the inflation response for alternative choices $\pi_0 = 0$, $\pi_0 = -0.25\%$, and $\pi_0 = -1\%$, also noted by triangular markers. Reading from top to bottom, a permanent fiscal surplus must rise by $1.06\%,\ 7.4\%,\ 13.7\%$, and 26.4% of the debt, with $r = 2\%$ and short-term debt.

Figure 4 presents the inflation response for the same model and interest-rate path as Figure 1, but for different values of initial inflation π_0. The solid line repeats the case $\pi_0 = -0.5\%$ from Figure 1. The dashed lines present choices $\pi_0 = 0$, $\pi_0 = -0.25\%$, and $\pi_0 = -1\%$.

Initial inflation, driven by the initial equilibrium-selection/fiscal shock, has a major influence on the path of inflation. With $\pi_0 = 0$, inflation rises throughout the episode. Producing disinflation depends crucially on a negative initial inflation shock. The standard Calvo model behaves the same way, but each inflation path converges to the interest rate path monotonically, without the period of amplification.

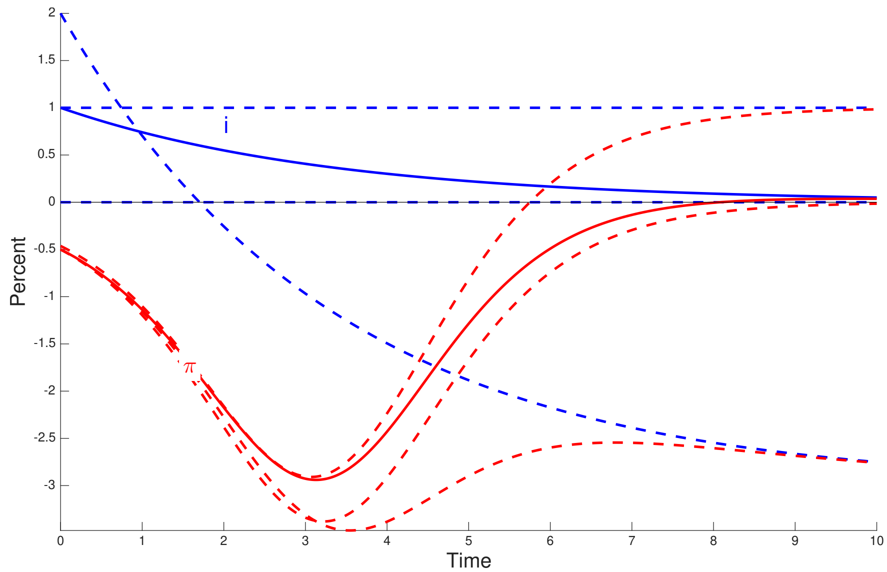

Figure 5: Inflation (red) for various values of the interest-rate path (blue), and the same initial inflation $\pi_0 = -0.5\%$. Continuous-time model with a standard dynamic IS curve and a generalized Lucas Phillips curve. Parameters $\sigma = 0.2$, $\kappa = 0.2$, $a = 1$, $\pi_0 = -0.5\%$. The permanent shock uses $\eta_i = 0$, other lines use $\eta_i = 0.3$. With $r = 2\%$, and short term debt, a permanent fiscal surplus must rise by 12.7%, 13.7%, 15.9%, and 11.0% of the debt, from top to bottom.

Figure 5 presents the inflation response for different values of the interest-rate path, holding constant initial inflation π_0. The solid line is the same case as other figures, with an AR(1) interest-rate path. The dashed lines use the indicated interest-rate paths. The initial disinflation is driven by the selection/fiscal shock π_0, as it is nearly the same for all interest-rate paths. Interest rates fully control long-run inflation however. Each inflation path ends up at its corresponding interest rate. You can tell them apart by reading from right to left.

I include a response with a permanent interest-rate rise. Inflation rises to meets this interest rate in the long run. This response emphasizes an unconventional but desirable property of rational expectations models, including the model with a Calvo Phillips curve. Inflation is stable and neutral in the long run. Stability and long-run neutrality imply that higher interest rates must eventually raise inflation, just as stability and long-run neutrality imply that higher money growth must eventually raise inflation in standard monetarist analysis. In this response, however, inflation declines in the short run. So common experience, which sees a lot more short runs than long runs, could well see that negative sign but not the long run positive relationship.

I include a response with no interest-rate movement at all. In fiscal theory, this is the response to a fiscal tightening equal to 15.9% of the value of debt, with no change in monetary policy. This is, in my interpretation (Cochrane, 2026a), just the kind of shock that occurred in 2021-2023 with the opposite sign. In a new-Keynesian interpretation, this is an "open-mouth" operation, a pure equilibrium-selection shock. The central bank announces a changed inflation target π^*, or a well-constructed sequence of Taylor-rule disturbances $\{u_t\}$, that generate a change of inflation with no movement in the interest rate. Fiscal policy then "passively" enacts the needed 15.9% fiscal tightening.

I also include an interest-rate path that rises temporarily to 2%, and then declines past its original point. This path is reminiscent of what happened in the 1980s, when a high interest rate was followed by lower rates as inflation came down. Cause and effect are reversed from the usual IS-LM or old-Keynesian interpretation, however. Here, the interest rates drag down inflation, not the other way around. Still, central banks usually raise rates to permanently lower inflation, so we should occasionally plot monetary policy responses that look something like successful disinflations, not just AR(1)s that leave no mark in the long run. It is comforting that this model can produce something that *looks* like the 1980s, though with a mechanism utterly different from the standard one.

5 Long-Term Debt and a Coordinating Mechanism

It seems that monetary policy relies on a coincident fiscal contraction for any disinflationary power of higher interest rates, with both Calvo and generalized Lucas Phillips curves. That isn't altogether unreasonable as a reading of the data. The same events that provoke a central bank to tighten also provoke fiscal authorities to tighten. But if the central bank's interest-rate increases are per se counterproductive, that is news to central banks and most economists.

The Taylor rule with AR(1) disturbance offers a vision linking higher interest rates to the fiscal tightening that does all the disinflation work. By pledging to follow that policy, not any of the other disturbance paths that produce the same interest-rate path, the central bank ties higher interest rates to contractionary equilibrium selection, and forces fiscal authorities to enact the necessary surpluses. But central banks do not follow explosive equilibrium-selection policies, and if they did the restriction to an AR(1) or other specific disturbance process is a tenuous link between interest rates and equilibrium selection.

Here I pursue the one coordination mechanism I know of in which higher interest rates lower initial inflation π_0, and thus subsequent inflation, without requiring any change in fiscal policy. This is the "stepping on a rake" (Sims, 2011; Cochrane, 2017b) or "unpleasant interest rate arithmetic" mechanism (Cochrane, 2023).

To make a calculation, I assume a geometric maturity structure in which the face value at maturity j is proportional to $\omega e^{-\omega j}$, and I assume the expectations hypothesis so the bond price is

$$q_0 = -\int_{t=0}^{\infty} e^{-(r+\omega)t} i_t dt.$$

Then the fiscal condition (23) becomes

$$\int_{t=0}^{\infty} e^{-rt} \tilde{s}_t = \int_{t=0}^{\infty} e^{-rt}(i_t - \pi_t)dt - p_0 - \int_{t=0}^{\infty} e^{-(r+\omega)t} i_t dt. \tag{24}$$

Given a maturity structure ω and a path of interest rates, we can then find the initial price level p_0 that results when there is no change in surpluses on the left-hand side. The Appendix gives formulas.

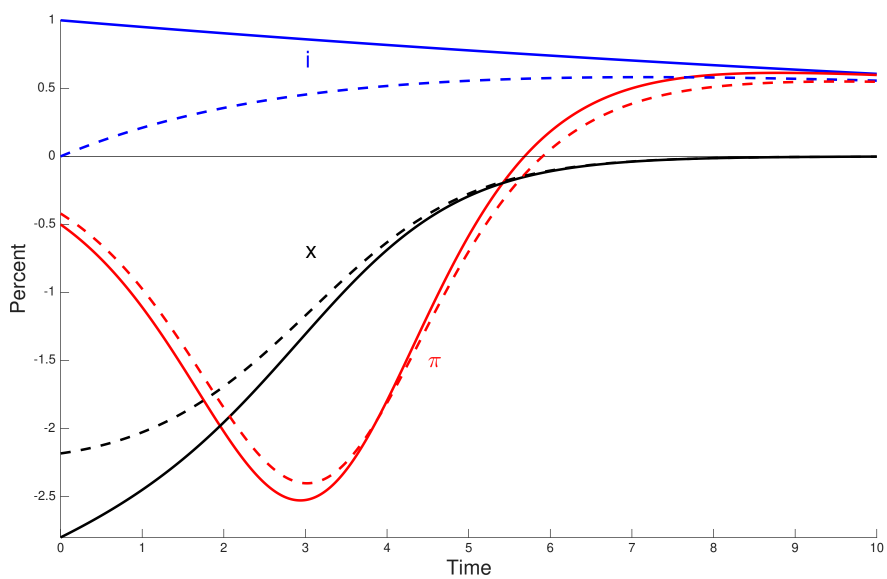

Figure 6: Higher interest rates can lower inflation with no fiscal tightening. Response of inflation and output to the indicated interest-rate path with no change in surpluses and long-term debt. Continuous-time model with a dynamic IS curve and a generalized Lucas Phillips curve. Parameters $\sigma = 0.2$, $\kappa = 0.2$, $a = 1$, and $\omega = 0.0026$. Solid lines: $i_t = e^{-0.05t}$. Dashed lines: $i_t = e^{-0.05t} - e^{-0.3t}$.

Figure 6 presents a calculation. This is the same model and parameters as in Figure 1. I reverse-engineer a maturity structure parameter $\omega = 0.0026$ which, with more persistent interest rate movement $\eta_i = 0.05$ rather than $\eta_i = 0.30$ generates $\pi_0 = -0.5\%$, to be consistent with previous graphs. The dynamics are not different for a given initial price level. The framework just gives a reason to tie the initial price level decline to rising interest rates.

The mechanism: When the central bank raises the nominal interest rate persistently, it lowers long-term bond prices and the nominal market value of government debt. With flexible prices, the real present value of surpluses does not change, so the price level must fall. With sticky prices, higher real interest rates initially lower the present value of surpluses. But those high real interest rates must come from lower inflation, so the price level falls slowly.

Higher interest rates all on their own now can lower inflation, at least temporarily. This mechanism gives central banks a reason to raise interest rates when they want to lower inflation. Conversely, lowering interest rates, which would otherwise lower inflation, temporarily raises inflation. That fact dissuades the central bank from that otherwise (in these models) attractive "neo-Fisherian" policy.

With a standard Calvo Phillips curve, this mechanism still generates inflation that jumps down and then rises going forward (Cochrane, 2026b). The generalized Lucas Phillips curve produces disinflation that increases going forward with this mechanism. It makes the disinflation larger than that initial jump, to the point that one might miss the jump empirically.

Of course, this mechanism, like the standard new-Keynesian model, has nothing to do with the standard old-Keynesian story that high real interest rates depress aggregate demand which pushes down inflation via the Phillips curve. But at least it gives a result with much the same appearance as the standard beliefs.

	Maturity ω, perpetual to instantaneous				
	0	0.0026	0.1	0.3	∞
s_0/r, $\eta_i = 0.05$ (Figure 6)	-0.50%	0	7.9%	12.9%	13.8%
s_0/r, $\eta_i = 0.30$ (Figure 1)	10.6%	10.6%	11.4%	12.1%	13.7%

Table 1: Surplus required for different values of the maturity structure of debt ω. This is the left-hand side of equation (24).

Table 1 presents the change in surplus, the left side of equation (24), required for the solution plotted in Figure 6, for different values of the maturity structure of debt ω. If debt has maturity structure characterized by $\omega = 0.0026$, this equilibrium requires exactly no change in surplus. This is the central case whose possibility the calculation emphasizes.

The table also presents the same calculation for the solution plotted in Figure 1. With the less persistent interest rate $\eta_i = 0.30$, the plotted solution requires a fiscal contraction for every debt maturity. To generate disinflation early on without a fiscal contraction, there must be inflation later on, as one sees in Figure 6. In Figure 1, the interest rate essentially returns to zero before inflation has a chance to follow interest rates upward. The long-term debt mechanism requires persistent interest rates to generate lower inflation without a fiscal contraction.

I emphasize the response with no change in current and future primary surpluses to illustrate its possibility, and to give a reason by which the central bank acting on its own might wish to raise interest rates in order to lower inflation, even temporarily. Other coordinated monetary-fiscal responses are interesting and important for actual policy and historical episodes. It is not necessary to hold surpluses constant, or to specify them exogenously, or to specify that they do not react to central bank action or to macroeconomic variables. Matching episodes or realistic policy evaluation should include how surpluses react to inflation, recession, interest costs, and debt, and consider coincident and coordinated fiscal policy shocks.

5.1 Forward Guidance and Inertial Interest Rates

VARs focus on the short-lived AR(1)-like interest rate excursions that typically follow small identified exogenous monetary policy shocks. The Fed, however, increasingly relies on forward guidance, promises of future interest rates with no immediate action. Also, when it turns to tighter or looser policy, it does not do so all at once but rather begins a predictable set of interest rate moves that can last a year or more. In most estimates, the Fed follows an "inertial" rule with past interest rates on the right-hand side, or a partial-adjustment rule, which display the same sort of persistence. These long-term movements dominate movements in interest rates, and we would like to know how inflation reacts to such movements.

The long-term debt mechanism gives a reason for both practices: The key to reducing inflation today is to raise expectations of high nominal interest rates and inflation in the future, to lower long-term bond prices and thus to lower near-term inflation. Higher short-run nominal interest rates are really irrelevant or even counterproductive to inflation reduction. In this feature, the stepping on a rake or unpleasant interest rate arithmetic mechanism differs fundamentally from the standard doctrine, which focuses on today's actual interest rates and their effect on aggregate demand.

To display this behavior, I feed the model a mixture of AR(1)s, which generates a hump-shaped interest-rate path,

$$i_t = i_s(e^{-\eta_1 t} - e^{-\eta_2 t}).$$

i_s is a scale variable, since $i_0 = 0$. This path can represent either a forward guidance announcement—interest rates will be higher or lower for a period of years in the future—or the beginnings of a tightening or loosening cycle. The solution is a linear combination of solutions we already have.

Figure 6 includes this calculation as well, represented in the dashed lines. I use the same maturity structure of the debt $\omega = 0.0026$, and I again hold surpluses fixed. The different interest-rate path gives a slightly different p_0 and π_0. The inflation and output paths are very similar to those of the previous case, even though the interest-rate paths are quite different. The contrast emphasizes that the *long-run* behavior of nominal rates is crucial to the long-term debt mechanism, and short-run behavior of interest rates is not important.

6 Downweighted Expectations and Habits

The dynamic paths of inflation and output depend on the IS part of the model as well as the Phillips curve. While still striving for the simplest textbook model, I investigate here two different and common IS specifications. Both allow a hump-shaped response of output as well as of inflation.

6.1 Downweighted Expectations

I consider an IS curve of the form

$$m\frac{dx_t}{dt} - (1-m)x_t = \sigma\left(i_t - \frac{dp_t}{dt}\right). \tag{25}$$

This specification allows an intermediate case between the previous dynamic IS curve, with $m = 1$, and a static IS curve in which the level of output is directly affected by the real interest rate, with $m = 0$. The specification (25) is the continuous-time equivalent of downweighting future output by a factor m in discrete time,

$$x_t = mx_{t+1} - \sigma(i - \pi),$$

as in Gabaix (2020).

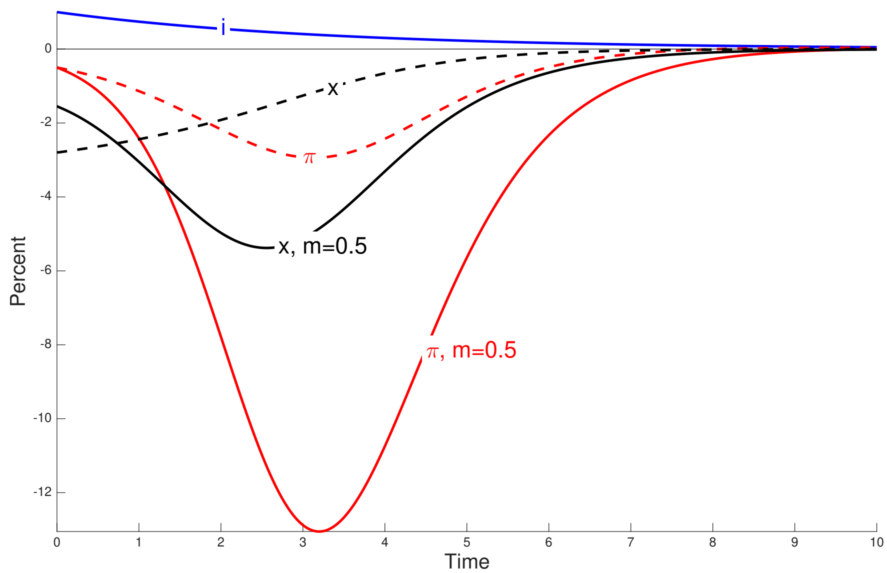

Figure 7: Inflation and output with a generalized Lucas Phillips curve and an IS curve with downweighted expectations. Parameters $\sigma = 0.2$, $\kappa = 0.2$, $a = 1$, $\eta_i = 0.3$, $i_0 = 1\%$, $\pi_0 = -0.5\%$, $r = 2\%$. Dashed lines are the previous case $m = 1$. Solid lines are downweighted with $m = 0.5$. The surplus rises by $\tilde{s}/r = 13\%$ and 43% of the value of debt in the two cases.

Figure (7) presents the results, contrasting the previous case ($m = 1$, dashed lines) and a downweighted case ($m = 0.5$, solid lines). Otherwise I use the same parameters.

The specification enhances the power of the Lucas Phillips curve to generate lower inflation. Equivalently, it lowers the size of the initial shock needed to get inflation going. It also leads to output that declines before rising again, bringing that response closer to common policy beliefs. Output still moves before inflation, as it does in most VARs.

6.2 Habits

I mix the generalized Lucas Phillips curve with an IS curve that features external habit persistence. This specification maintains the purity of rational expectations. It is a well-known ingredient to produce hump-shaped output dynamics.

Marginal utility depends on consumption relative to a moving average of past consumption. The IS curve becomes

$$E_t d\left(x_t - \theta b \int_{s=0}^{\infty} e^{-bs} x_{t-s} ds\right) = \sigma[i_t dt - E_t(dp_t)]. \tag{26}$$

(In discrete time, one-period habits are common, $x_t - \theta x_{t-1}$, but adding a cost to a second derivative of consumption is awkward in continuous time. This AR(1) formulation keeps the consumption lag in place while shrinking the time period.)

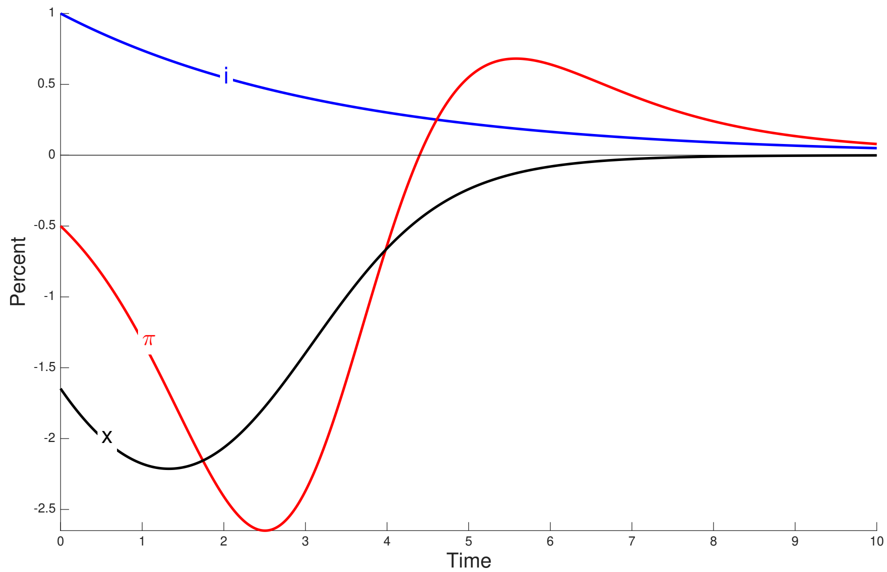

Figure 8: Inflation and Output Response with Autoregressive Habit and Generalized Lucas Phillips Curve. Parameters $\sigma = 0.2$, $\kappa = 0.2$, $a = 1$, $\eta_i = 0.3$, $i_0 = 1\%$, $\pi_0 = -0.5\%$, $r = 2\%$, $\theta = 0.7$, $b = 1$. The surplus rises by $\tilde{s}/r = 8.23\%$ of the value of debt.

Figure 8 presents the results, adding habit with $b = 1$ and $\theta = 0.7$. Habits induce a hump-shaped output response, as hoped.

To derive the inflation response to interest rates in this case, define the external habit as

$$X_t = b \int_{s=0}^{t} e^{-b(t-s)} x_s ds$$

which solves

$$\frac{dX_t}{dt} = b(x_t - X_t).$$

Then (26) becomes

$$\frac{dx}{dt} - \theta b(x_t - X_t) = \sigma \left(i_t - \frac{dp}{dt} \right).$$

Eliminate output and its derivative from the generalized Lucas Phillips curve with exponential weights (8) and its derivative (9), yielding

$$(e^{-at} + \sigma\kappa)\frac{dp}{dt} - (a + \theta b)e^{-at} p_t + \theta b \kappa X_t = \sigma \kappa i_t.$$

$$\frac{d\kappa X_t}{dt} = b(\kappa e^{-at}p_t - \kappa X_t).$$

Solve this pair of differential equations numerically, starting with the initial condition

$$(1+\sigma\kappa)\pi_0 - (a+\theta b)p_0 = \sigma\kappa i_0$$

and $X_0 = 0$.

Perhaps habits alone can provide the hump-shaped inflation response we are looking for, without the generalized Lucas Phillips curve? After all, habits are a standard ingredient to induce hump shapes. It turns out that habits can induce a hump shape in *output* but not one in inflation.

To investigate, I solve a model with habit IS and standard Calvo Phillips curve. The response solves

$$\frac{dX}{dt} = b(x_t - X_t)$$

$$\frac{dx_t}{dt} - \theta b(x_t - X_t) = \sigma(i_t - \pi_t)$$

$$\frac{d\pi_t}{dt} = r\pi_t - \kappa x_t$$

$$\frac{di}{dt} = -\eta_i i_t$$

The initial condition is given by $X_0 = 0$, π_0, and i_0. I use the one forward-looking eigenvalue to find x_0. I solve the model by the standard matrix method, reviewed in the Appendix .

Figure 9 presents the results. No, at least at the parameter values we have been using so far, habits alone together with the standard forward-looking Phillips curve do not generate a hump-shaped inflation response.

7 Lagged Inflation in the Phillips Curve

Lagged inflation in the Phillips curve is the leading contender in the literature for a single modification to produce hump-shaped inflation. Steinsson (2003) and Galí and Gertler (1999) focus on it, and it is an ingredient of larger models including Christiano, Eichenbaum, and Trabandt (2016, 2018) and Smets and Wouters (2007). A variety of microfoundations have been adduced, including price indexation between Calvo resets and rule-of-thumb price setters. The Appendix

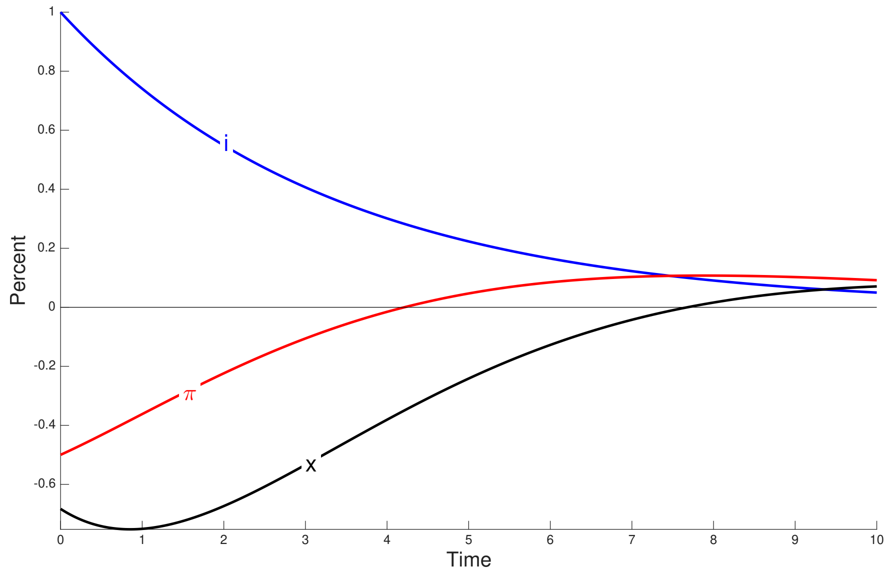

Figure 9: Autogressive Habits and Calvo Phillips Curve. Parameters $\theta = 0.7$, $b = 1$, $\pi_0 = -0.5\%$, $\eta_i = 0.3$, $\phi = 1.1$, $r = 2\%$, $\kappa = 0.2$, $\sigma = 0.2$, $i_0 = 1\%$.

includes a review. However, I have not found an investigation close enough to the treatment of the generalized Lucas curve here, so I present that comparison.

Phillips curves with mixed leads and lags of inflation do not port easily to continuous time. For that reason, and since my conclusions are negative, I present its effect in discrete time as the other papers of the literature have done. I investigate

$$
\begin{aligned}
x_t &= E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1}) \\
\pi_t &= (1-\gamma) E_t \pi_{t+1} + \gamma \pi_{t-1} + \kappa x_t .
\end{aligned}
$$

Eliminating x_t and solving the resulting difference equation, we obtain

$$
\begin{aligned}
&\pi_{t+1} = C\lambda_1^{t+1} + \frac{(1-\lambda_1)(1-\lambda_2)(1-\lambda_3)}{\lambda_2 - \lambda_3} \times \\
&\times \left[\frac{\lambda_2}{1-\lambda_1\lambda_2} \left(\sum_{j=0}^{\infty} \lambda_1^j i_{t+j} + \sum_{j=1}^{\infty} \lambda_2^j i_{t-j} \right) - \frac{\lambda_3}{1-\lambda_1\lambda_3} \left(\sum_{j=0}^{\infty} \lambda_1^j i_{t+j} + \sum_{j=1}^{\infty} \lambda_3^j i_{t-j} \right) \right] \qquad (27)
\end{aligned}
$$

where $|\lambda_i| < 1$ are the roots of the third-order lag polynomial, found numerically. This is a natural generalization of the formula for a forward-looking Phillips curve, (21).

For an AR(1) interest rate $i_t = i_0 \eta_i^t$ for $t \geq 0$ with a surprise at $t = 0$, this formula becomes

$$\pi_{t+1} = C\lambda_1^{t+1} + i_0 \frac{(1-\lambda_1)(1-\lambda_2)(1-\lambda_3)}{\lambda_2 - \lambda_3} \times$$
$$\times \left[\frac{\lambda_2}{\eta_i - \lambda_2} \left(\frac{\eta_i^{t+1}}{1-\lambda_1\eta_i} - \frac{\lambda_2^{t+1}}{1-\lambda_1\lambda_2} \right) - \frac{\lambda_3}{\eta_i - \lambda_3} \left(\frac{\eta_i^{t+1}}{1-\lambda_1\eta_i} - \frac{\lambda_3^{t+1}}{1-\lambda_1\lambda_3} \right) \right]. \tag{28}$$

I present the algebra in the Appendix.

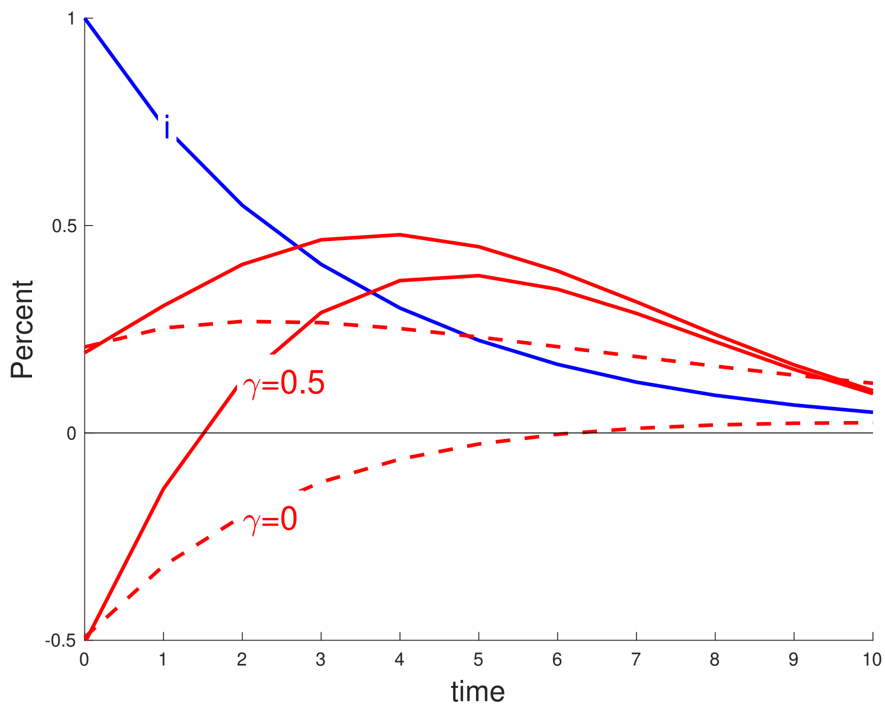

Figure 10: Lagged inflation in the Phillips curve, $\pi_t = (1-\gamma)E_t\pi_{t+1} + \gamma\pi_{t-1} + \kappa x_t$ plus the intertemporal IS curve. The top curves uses $C = 0$, the lower curves use $C = -0.75$. Solid lines are $\gamma = 0.5$, dashed lines are $\gamma = 0$. Parameters $\sigma = 0.2$, $\kappa = 0.2$, $\eta_i = e^{-0.3} = 0.74$.

Figure 10 presents a calculation, using $\gamma = 0.5$ (solid) with the standard case $\gamma = 0$ (dash) as a reference. The higher set of curves presents the $C = 0$ case. Already, the $\gamma = 0$ classic case shows a slight hump shape. It is the sum of two AR(1)s, the interest rate plus the root of the lag polynomial. Moving to $\gamma = 0.5$, we now have the difference between two such terms, which produces more pronounced hump-shaped dynamics as advertised.

Nonetheless, you can see both in the formula and in the graph that higher nominal interest rates still raise inflation. The hump shapes go the “wrong” way. In each term of (27), inflation is a moving average of the interest rate with positive coefficients. The second term enters with a negative sign, so negative inflation is possible. But the sum of all coefficients of inflation on interest rate is 1. This model is long-run neutral by construction. Larger values of γ produce inflation that rises even further and for longer, before converging in a wavy (complex roots) pattern. No weight $\gamma \in (0, 1)$ produces inflation that falls in the first few years.

In the lower set of curves, I add an equilibrium-selection transient, $C = -0.75$, chosen to produce approximately $\pi_0 = -0.5$ consistent with prior calculations. As you might suspect from the formulas, this change drags initial inflation down, but inflation then rises as it does in the standard Calvo model. The lag in the Phillips curve adds a hump shape to this response as well, but inflation is higher at all horizons than is the original case. (Readers who prefer a Taylor rule disturbance may visualize $u_t = i_t - \phi\pi_t$ to select this equilibrium in place of the assumption on C.)

In sum, lags in the Phillips curve do not produce inflation that declines following an interest rate rise, either on their own (top lines), nor by amplifying an initial equilibrium-selection shock. Though they induce hump-shaped dynamics, they induce those dynamics in the wrong direction.

The standard doctrine, that inflation does not move on impact and then falls, can be represented easily with a fully backward-looking model. (See the Appendix and Cochrane (2024).) This observation may have motivated authors to add backward-looking elements to the Phillips curve. But the response to interest rates depends on the whole model, not just the Phillips curve. My experiment above keeps forward-looking expectations in the real interest rate $i_t - E_t\pi_{t+1}$ of the IS curve. Examining the Phillips curve alone is a worthy enterprise, but lags in the Phillips curve alone do not necessarily produce lags in the entire model.

8 Concluding Comments

This paper investigates a generalization of the classic Lucas Phillips curve. It works towards a model of textbook simplicity and analytic tractability that can start to bridge the yawning gap between the effects of monetary policy most economists and central bank officials talk about and the effects of monetary policy in rational expectations/DSGE/new-Keynesian models that have dominated academic work for the last three decades, without adding a smorgasbord of complications. In the standard doctrine, higher nominal interest rates mean higher real interest rates that lower aggregate demand, lower output, and then via a Phillips curve lower inflation, all pushing inflation and output downward in the future, with long and perhaps variable lags. That view is captured by 1970s-era adaptive expectations models, but not by models with forward-looking expectations (Cochrane, 2024). In rational-expectations models, higher nominal rates raise inflation going forward. The only mechanism for lower inflation is a coincident downward equilibrium-selection/fiscal shock.

The generalized Lucas Phillips curve turns a small downward inflation or price-level jump into a larger disinflation, generating the basic sign of standard beliefs. The long-term debt unpleasant-arithmetic mechanism allows higher nominal interest rates to set off such a small downward jump without a coincident fiscal contraction, and it gives central banks a reason to raise interest rates if they want to lower inflation.

The resulting responses look a great deal more like those of common beliefs than the responses of the standard model with a forward-looking Calvo Phillips curve. The responses continue to act, however, by a completely different mechanism, as do those of all other rational expectations models.

I pursue here a standard research paradigm: I modify the baseline new-Keynesian model to aim for results that look like the results of standard beliefs, accepting that those results are produced by an utterly different mechanism. Alternatively, the standard beliefs may be wrong. VARs are imprecisely measured (Ramey, 2016), subject to a long specification search to produce the "right" answer (Enzinger et al., 2026), and often measure the wrong thing: They do not tell us how the persistent interest rate movements that central banks usually follow would affect output and other variables; they do not tell us how changes in the policy rule would affect the economy; and nobody has tried to construct monetary policy shocks orthogonal to fiscal policy to measure what central banks can do on their own. Maybe monetary policy does act quickly (Buda et al., 2023). It is hard to measure instantaneous inflation, and the change from a year ago will always appear sluggish. Many VARs do not allow inflation to jump on impact by assumption, either by orthogonalization or by use of high-frequency shock data and lower frequency inflation data. Miranda-Agrippino and Ricco (2021) show that when one removes that restriction, inflation jumps. But seeing if the facts conform to the theory is a different path from seeing if the theory can be made to conform somewhat to the currently-perceived facts.

Declaration of generative AI and AI-assisted technologies in the manuscript preparation process:

During the preparation of this work the author used claude and refine in order to find typos and other errors. After using this tool/service, the author reviewed and edited the content as needed and takes full responsibility for the content of the published article.

References

Angeletos, George-Marios and Zhen Huo. 2021. "Myopia and Anchoring." *American Economic Review* 111 (4):1166–1200. URL https://www.aeaweb.org/articles?id=10.1257/aer.20191436.

Auclert, Adrien, Matthew Rognlie, and Ludwig Straub. 2020. "Micro Jumps, Macro Humps: Monetary Policy and Business Cycles in an Estimated HANK Model." Manuscript.

Ball, Laurence. 1994. "Credible Disinflation with Staggered Price-Setting." *The American Economic Review* 84 (1):282–289. URL http://www.jstor.org/stable/2117983.

Buda, Gergely, Vasco Carvalho, Giancarlo Corsetti, Jo ao B. Duarte, Stephen Hansen, Alfonso S. Moura, Álvaro Ortiz, Tomasa Rodrigo, José V Rodríguez Mora, and Guilherme Alves da Silva. 2023. "The Short Lags of Monetary Policy." *CEPR Discussion Paper DP18022* URL https://cepr.org/publications/dp18022.

Calvo, Guillermo A. 1983. "Staggered prices in a utility-maximizing framework." *Journal of Monetary Economics* 12 (3):383–398. URL https://www.sciencedirect.com/science/article/pii/0304393283900600.

Carroll, Christopher D., Edmund Crawley, Jiri Slacalek, Kiichi Tokuoka, and Matthew N. White. 2020. "Sticky Expectations and Consumption Dynamics." *American Economic Journal: Macroeconomics* 12 (3):40–76. URL https://www.aeaweb.org/articles?id=10.1257/mac.20180286.

Choi, Chek, Marcus Hagedorn, Juan Llavador, and Kurt Mitman. 2025. "Inflation Persistence and a new Phillips Curve." *Manuscript* URL https://hkchekc.github.io/assets/doc/Phillips_Curve_draft.pdf.

Christiano, Lawrence J., Martin Eichenbaum, and Charles Evans. 2005. "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy." *Journal of Political Economy* 113:1–45.

Christiano, Lawrence J., Martin S. Eichenbaum, and Mathias Trabandt. 2016. "Unemployment and Business Cycles." *Econometrica* 84 (4):1523–1569.

———. 2018. "On DSGE Models." *Journal of Economic Perspectives* 32 (3):113–40. URL https://www.aeaweb.org/articles?id=10.1257/jep.32.3.113.

Cochrane, John H. 2017a. "The New-Keynesian Liquidity Trap." *Journal of Monetary Economics* 92:47–63.

———. 2017b. “Stepping on a Rake: The Fiscal Theory of Monetary Policy.” *European Economic Review* 101:354–375.

———. 2023. *The Fiscal Theory of the Price Level*. Princeton NJ: Princeton University Press.

———. 2024. “Expectations and the Neutrality of Interest Rates.” *Review of Economic Dynamics* 53:194–223. URL https://www.sciencedirect.com/science/article/pii/S1094202524000140.

———. 2026a. *Inflation*. MIT Press. URL https://www.johnhcochrane.com/research-all/inflation. Forthcoming.

———. 2026b. “New-Keynesian Inflation Control is Equilibrium Selection.” *Manuscript, Hoover Institution* .

Coibion, Olivier. 2010. “Testing the Sticky Information Phillips Curve.” *The Review of Economics and Statistics* 92 (1):87–101. URL https://doi.org/10.1162/rest.2009.11148.

Dupor, Bill. 2023. “Examining Long and Variable Lags in Monetary Policy.” *Regional Economist* URL https://www.stlouisfed.org/publications/regional-economist/2023/may/examining-long-variable-lags-monetary-policy. May.

Dupor, Bill, Tomiyuki Kitamura, and Takayuki Tsuruga. 2010. “Integrating Sticky Prices and Sticky Information.” *The Review of Economics and Statistics* 92 (3):657–669. URL https://doi.org/10.1162/REST_a_00017.

Enzinger, Matthias, Sebastian Gechert, Philipp Heimberger, Franz Prante, and Daniel F. Romero. 2026. “The overstated effects of conventional monetary policy on output and prices.” *OSF Preprint* URL https://osf.io/preprints/osf/72cen_v2.

Friedman, Milton. 1948. “A Monetary and Fiscal Framework for Economic Stability.” *The American Economic Review* 38 (3):245–264.

———. 1968. “The Role of Monetary Policy.” *The American Economic Review* 58:1–17.

Gabaix, Xavier. 2020. “A Behavioral New Keynesian Model.” *American Economic Review* 110:2271–2327.

Galí, Jordi and Mark Gertler. 1999. “Inflation dynamics: A structural econometric analysis.” *Journal of Monetary Economics* 44 (2):195–222. URL https://www.sciencedirect.com/science/article/pii/S0304393299000239.

García-Schmidt, Mariana and Michael Woodford. 2019. "Are Low Interest Rates Deflationary? A Paradox of Perfect-Foresight Analysis." *American Economic Review* 109:86–120.

González, Ignacio and Vasudeva Ramaswamy. 2024. "Capacity Utilization, Markup Cyclicality, and Inflation Dynamics." *Manuscript* URL https://vasudeva-ram.github.io/research/.

Kiley, Michael T. 2007. "A Quantitative Comparison of Sticky-Price and Sticky-Information Models of Price Setting." *Journal of Money, Credit and Banking* 39 (s1):101–125. URL https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1538-4616.2007.00017.x.

King, Robert G. 2000. "The New IS-LM Model: Language, Logic, and Limits." *Federal Reserve Bank of Richmond Economic Quarterly* 86 (3):45–104.

Klenow, Peter J. and Jonathan L. Willis. 2007. "Sticky information and sticky prices." *Journal of Monetary Economics* 54:79–99. URL https://www.sciencedirect.com/science/article/pii/S0304393207000542. Supplement issue: October 20-21 2006 Research Conference on 'Microeconomic Adjustment and Macroeconomic Dynamics' Sponsored by the Swiss National Bank (http://www.snb.ch) and Study Center Gerzensee (www.szgerzensee.ch).

Kwicklis, Noah. 2025. "Sticky Expectations, Fiscal Transfers, Inflation, and Unemployment in HANK." *Manuscript* URL https://noahkwicklis.com/workingpapers/KwicklisJMP.pdf.

Lucas, Robert E. 1972. "Expectations and the Neutrality of Money." *Journal of Economic Theory* 4:103–124.

———. 1973. "Some International Evidence on Output-Inflation Tradeoffs." *The American Economic Review* 63 (3):326–334.

Maćkowiak, Bartosz, Filip Matějka, and Mirko Wiederholt. 2023. "Rational Inattention: A Review." *The Journal of Economic Literature* 61:226–273.

Maćkowiak, Bartosz and Mirko Wiederholt. 2009. "Optimal Sticky Prices under Rational Inattention." *American Economic Review* 99:769–803.

———. 2015. "Business Cycle Dynamics under Rational Inattention." *The Review of Economic Studies* 82 (4):1502–1532. URL https://doi.org/10.1093/restud/rdv027.

Mankiw, N. Gregory and Ricardo Reis. 2002. "Sticky Information versus Sticky Prices: A Proposal to Replace the New Keynesian Phillips Curve." *Quarterly Journal of Economics* 117:1295–1328.

———. 2007. “Sticky Information in General Equilibrium.” *Journal of the European Economic Association* 5 (2-3):603–613. URL https://doi.org/10.1162/jeea.2007.5.2-3.603.

Miranda-Agrippino, Silvia and Giovanni Ricco. 2021. “The Transmission of Monetary Policy Shocks.” *American Economic Journal: Macroeconomics* 13 (3):74–107.

Nelson, Edward. 2020. *Milton Friedman and Economic Debate in the United States, 1932–1972, Volume 1*. Chicago: University of Chicago Press.

Nimark, Kristoffer. 2008. “Dynamic pricing and imperfect common knowledge.” *Journal of Monetary Economics* 55 (2):365–382. URL https://www.sciencedirect.com/science/article/pii/S0304393207001614.

Ramey, Valerie. 2016. “Macroeconomic Shocks and Their Propagation.” In *Handbook of Macroeconomics Vol. 2*, edited by John B. Taylor and Harald Uhlig. Amsterdam: Elsevier, 71–162.

Reis, Ricardo. 2006. “Inattentive consumers.” *Journal of Monetary Economics* 53 (8):1761–1800. URL https://www.sciencedirect.com/science/article/pii/S0304393206001401.

Rotemberg, Julio J. 1982. “Sticky Prices in the United States.” *Journal of Political Economy* 90 (6):1187–1211.

Sims, Christopher A. 2003. “Implications of rational inattention.” *Journal of Monetary Economics* 50 (3):665–690. URL https://www.sciencedirect.com/science/article/pii/S0304393203000291. Swiss National Bank/Study Center Gerzensee Conference on Monetary Policy under Incomplete Information.

———. 2011. “Stepping on a Rake: The Role of Fiscal Policy in the Inflation of the 1970s.” *European Economic Review* 55:48–56.

Smets, Frank and Raf Wouters. 2007. “Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach.” *American Economic Review* 97:586–606.

Steinsson, Jón. 2003. “Optimal monetary policy in an economy with inflation persistence.” *Journal of Monetary Economics* 50 (7):1425–1456. URL https://www.sciencedirect.com/science/article/pii/S0304393203000953.

Svensson, Lars E. O. 2003. “Comment on Michael Woodford, ‘Imperfect Common Knowledge and the Effects of Monetary Policy’.” In *Knowledge, Information and Expectations in Modern Macroeconomics: In Honor of Edmund S. Phelps*, edited by Philippe Aghion, Roman Frydman, Joseph Stiglitz, and Michael Woodford. Princeton: Princeton University Press, 59–63. URL https://larseosvensson.se/files/papers/woo110.pdf.

Verona, Fabio and Maik H. Wolters. 2013. "Sticky Information Models in Dynare." *Manuscript* URL https://www.dynare.org/wp-repo/dynarewp011.pdf.

Woodford, Michael. 2003a. "Imperfect common knowledge and the effects of monetary policy." In *Knowledge, Information, and Expectations in Modern Macroeconomics: In honor of Edmund S. Phelps*, edited by Philippe Aghion, Roman Frydman, Joseph Stiglitz, and Michael Woodford. Princeton: Princeton University Press, 22–58.

———. 2003b. *Interest and Prices*. Princeton: Princeton University Press.

Appendices

A Literature and its Lessons

The sticky information idea and attempts to modify the standard new-Keyensian model to produce inflation and output that drift down with higher interest rates rather than jump down and then rise are both well established. This paper's main contribution is a a simple and analytically tractable model, along with clarity on the link between interest rates and equilibrium selection.

A.1 Mankiw and Reis

In the current literature, the generalized Lucas Phillips curve is closest to the "sticky information" Phillips curve of Mankiw and Reis (2002, 2007).

Mankiw and Reis show that the sticky-information Phillips curve can display a hump-shaped inflation response. Mankiw and Reis (2002) (Figure I, bottom panel) show a pattern similar to that of my Figure 1, in response to a one-time decline in the money stock. Inflation jumps down in the first period, and then keeps going, before turning around and returning to zero. One might call that "locally explosive" as I do, or "inertial" as Mankiw and Reis do. That behavior contrasts with the forward-looking Phillips curve in the standard model completed with an IS curve and an interest-rate rule, in which inflation jumps down and then reverts in an AR(1) pattern.

Mankiw and Reis start from firms that wish to set their prices p_t^* by

$$p_t^* = p_t + \kappa x_t \tag{29}$$

where p_t is the overall price level and x_t is log aggregate output (my notation). Firm j that "updated its plans j periods ago" sets price

$$p_{j,t} = E_{t-j} p_t^*.$$

As I do, Mankiw and Reis leave out any information the firm might glean about the aggregate price level from information between $t-j$ and t. They additionally assume that the firm pays no attention to idiosyncratic demand. They specify that that the firm does not change price at all between $t-j$ and t. ("Plans" vs. "information" is important.) Averaging across firms in the economy, they obtain, in my notation,

$$p_t = \sum_{j=0}^{\infty} \alpha_j p_{j,t} = \sum_{j=0}^{\infty} \alpha_j E_{t-j}(p_t + \kappa x_t) \tag{30}$$

The differences are small. I place individual rather than aggregate output in (29), and I allow firms to change price and output at every date with only the expectations being sticky. Mankiw and Reis's specification leads to lagged expectations of output in (30) instead of current output, and lagged expectations of current output growth when they express their Phillips curve in terms of inflation. In turn, that complication makes it difficult to compute model solutions. The differences are not fundamental however. The starting point of this paper may be seen as a formulation of similar ideas in a more tractable form.

Why is the sticky-information Phillips curve not already widely used in subsequent models, and already a standard ingredient? Following standard practice, Mankiw and Reis (2002) derive a Phillips curve for inflation (p. 1300)

$$\pi_t = \left[\frac{\alpha\lambda}{1-\lambda}\right] x_t + \lambda \sum_{j=0}^{\infty} (1-\lambda)^j E_{t-1-j} \left(\pi_t + \alpha \Delta x_t\right). \tag{31}$$

The presence of the extra Δx_t on the right hand side is a nuisance. More importantly, it is not easy to embed this Phillips curve in a standard model and compute responses. Mankiw and Reis (2002) only draw illustrative responses to money supply shocks.

Mankiw and Reis (2007) take on the challenge of embedding their sticky information Phillips curve in a full model with an interest-rate target. Their aggregate log-linearized Phillips curve is (p.605)

$$p_t = \lambda \sum_{j=0}^{\infty} (1-\lambda)^j E_{t-j} \left[p_t + \frac{\beta(w_t - p_t) + (1-\beta)x_t - a_t}{\beta + \nu(1-\beta)} - \frac{\beta \nu_t}{(\nu - 1)[\beta + \nu(1-\beta)]} \right]$$

where beyond standard symbols, β measures returns to scale, a_t is productivity, and ν_t is a time-varying substitution elasticity between varieties. With wages in the Phillips curve, we also need a wage adjustment equation (p. 606)

$$w_t = \omega \sum_{j=0}^{\infty} (1-\omega)^j E_{t-j} \left[p_t + \frac{\gamma(w_t - p_t)}{\gamma + \psi} + \frac{l_t}{\gamma + \psi} + \frac{\psi(x_{\infty}^n - \theta R_t)}{\theta(\gamma + \psi)} - \frac{\psi \gamma_t}{(\gamma + \psi)(\gamma - 1)} \right]$$

where beyond standard symbols, l_t is labor, ψ is the Frisch elasticity of labor supply, θ is the intertemporal elasticity of substitution, and ω is the fraction of workers who become informed each period. Mankiw and Reis complete the model with a standard production function $x_t = a_t + \beta l_t$ and IS curve. They plot impulse-response functions to shocks (p.608). Sadly they omit the interest rate response, so we cannot relate inflation to the behavior of interest rates, which is the central question for monetary policy.

The Mankiw-Reis model "raises several thorny technical issues" (p. 603). "Ready-to-use solution algorithms" are not "particularly useful to solve the sticky-information model. The model involves both an infinite number of past expectations of the present through sticky information, as well as present expectations of variables at an infinite number of future dates through intertemporal smoothing. This double infinity implies that the state space of the model has an infinite dimension, which current algorithms cannot handle." (p. 604.) Mankiw and Reis provide an alternative solution algorithm, but few have followed them down that path. Verona and Wolters (2013) show how to compute approximate solutions in Dynare, but still few have followed.

I provide this level of detail so we can answer a puzzle, and learn appropriate lessons. Mankiw and Reis (2002) has, as of February 18, 2025, an impressive 3508 Google scholar citations. Mankiw and Reis (2007) though more directly useful to the construction of a standard NK-DSGE model with an interest-rate target, has only 309 citations. And of these, I have not yet found anyone who has *used* the Mankiw-Reis model. Claude.ai reports "What I can say with confidence is that while the Mankiw-Reis model has been influential in theoretical discussions, empirical applications using their exact framework have been less common than papers that cite or discuss their work conceptually." Mankiw and Reis (2007) may also have been overlooked because they jumped to the construction of a "medium-scale" model without a simpler "textbook" model.

Lucas (1972) has been even more influential, of course, with 8416 Google Scholar cites. Lucas's actual model is much harder. But the loglinear simplified form in Lucas (1973) led to a very simple and usable aggregate Phillips curve,

$$\kappa x_t = \pi_t - E_{t-1}\pi_t. \tag{32}$$

This form, rather than the full microfounded islands plus overlapping generations model, took off. (Actually, Lucas (1973) p. 327 (3) writes output of firm z with a lagged output term, as

$$y_{ct}(z) = \gamma[P_t(z) - E(P_t|I_t(z))] + \lambda y_{c,t-1}(z).$$

The lagged term appears to be ad-hoc, to give the model some output persistence, and is usually dropped in later literature.)

Calvo (1983) (13454 Google scholar citations) is similarly influential, despite its obviously counterfactual microfoundations, and the rather tortured path one must take to arrive at its familiar Phillips curve in terms of inflation rather than the price level (Woodford, 2003b, Ch. 3). But that Phillips curve is both intuitive and easy to incorporate into other models. Rotemberg (1982) quadratic menu costs offer a similar path to about the same aggregate Phillips curve as the Calvo model. The Rotemberg version does not generate price dispersion across firms, and thus different welfare analysis.

There are hundreds of other papers that investigate various microfounded models of sticky prices, many designed to replicate the equally large microeconomic empirical work on price stickiness. And yet they have not become standard ingredients of the larger project, or made their way to the standard textbook model.

The lesson seems clear: To be adopted by later researchers, a model must produce an easily *usable* and interpretable aggregate Phillips curve, or a similarly simple and tractable description of price-level dynamics. That principle guides this paper.

A.2 Medium-scale new-Keynesian Models

It is hardly news that the standard three-equation new-Keynesian model produces inflation and output that jump down and then rise, rather than slowly decline. A large number of models include a smorgasbord of additional ingredients to match VARs and thus produce something like the standard dynamics, though also not by the standard mechanism. Christiano, Eichenbaum, and Evans (2005) and Christiano, Eichenbaum, and Trabandt (2016, 2018) are notable examples. Many other empirically-oriented models, such as the famous Smets and Wouters (2007) model, fit well other data but still produce responses in which inflation jumps down with the shock and then rises after only a brief hump shape. (Smets and Wouters Figure 6, bottom left panel.) González and Ramaswamy (2024) obtain a hump-shaped inflation response by adding variable capacity utilization.

However, these models include major changes and elaborations of the basic model, including prices and wages fixed one period in advance, a strong one-period habit in utility, sticky wages driven by labor monopoly power, firms that must borrow to pay the next period's wage bill, capital with adjustment costs that depend on investment growth rather than the investment to capital ratio, and more. A common theme is lagged terms in the Phillips curve, IS curve, and investment equation, as well as specifications that add a second derivative. Where the old IS curve related the level of output to the real interest rate, and the new-Keynesian IS curve relates the growth rate of output to the real interest rate, a strong habit relates the growth of the growth of output to the real interest rate. Adjustment costs that depend on investment growth work the same way. Not all of these elaborations are consistent with microeconomic evidence. They are admirably reverse-engineered to produce aggregate impulse response functions that match the data, at the cost of some Lucas-critique robustness. They are also designed to match dynamics of a wide variety of series, with more emphasis on output, consumption, investment, and employment than on the price level. Inflation moves little in these models, as in the VARs that they match. They are really models of recessions, not minimal models of inflation.

For many applications, such extensions are reasonable and desirable. These papers' fame is justified. Policy applications often want a good fit to recent data, and to impulse-responses functions of many variables to many shocks, at the potential expense of simplicity, transparency, and Lucas-critique purity. But once again, my goal is to find a minimal model that captures only a hump-shaped inflation response to interest rates.

If the three-equation model captured the basic sign and operation of monetary policy, large-scale extensions, to accurately fit many time series and make quantitative policy evaluation, would make sense, just as the large scale Keynesian models of the late 1970s expanded on the basic ISLM model.

But the basic three-equation model does not capture what most economists and policy makers consider the basic sign and operation of monetary policy. Thus, such extensions leave open the question, what is the minimal *necessary* set of economically plausible ingredients that describe the basic sign and mechanism of monetary policy? What distinguishes the basic sign and operation of monetary policy from the flexible price rational expectations model $i_t = r + E_t\pi_{t+1}$? Which of these ingredients, if abandoned, would leave us with the uncomfortable implications of the flexible price model – higher interest rates raise inflation, period? And all economic models are, in the end, quantitative parables. Numerically-solved medium-scale models that operate with fundamentally different signs from the interpretable baseline leave open the question, just how do they work? My goal and contribution is focus on these questions.

A.3 Lags in the Phillips Curve

Of course, if one wishes to capture the intuition and mechanism of the standard doctrine, one can quickly write down an old-style ad-hoc ISLM model that does so. Lagged inflation in the Phillips and IS curves are the key, which are traditionally motivated by adaptive expectations. This is easy to see with a static IS curve. (See Cochrane, 2024.) Consider

$$x_t = -\sigma(i_t - \pi_{t-1})$$
$$\pi_t = \pi_{t-1} + \kappa x_t.$$

In this example, we have

$$\pi_t = (1 + \sigma\kappa)\pi_{t-1} - \sigma\kappa i_t.$$

Higher interest rates lower inflation going forward. Inflation is unstable under an interest rate peg; a Taylor rule $i_t = \phi\pi_t + u_t$ restores stability. The continuous-time version is

$$x_t = -\sigma(i_t - \pi_t)$$
$$\frac{d\pi_t}{dt} = \kappa x_t.$$

Adaptive expectations changes the sign on the right hand side of the Phillips curve, and specifies that π_t cannot jump. Now higher output means higher, not lower, inflation growth. We still have

$$\frac{d\pi_t}{dt} = -\sigma\kappa(i_t - \pi_t).$$

Higher interest rates do not move inflation instantly, and lower inflation going forward. Like the rest of the literature I have avoided this retreat to 1970s macroeconomics, trying to see if we can find a rational-expectations, optimizing model that gives the basic sign and operation of monetary policy.

Backward-looking terms in the Phillips curve, such as

$$\pi_t = (1 - \gamma)E_t\pi_{t+1} + \gamma\pi_{t-1} + \kappa x_t$$

are thus a natural hope to produce something like the traditional dynamics. I showed above that such backward-looking terms alone do not necessarily or easily produce a response in which higher interest rates lower inflation going forward. However, they are a common ingredient in more complex models that do so.

Lagged inflation in the Phillips curve also is a natural response to Ball (1994), who notes that with the standard Calvo model, inflation increases – inflation today less than future inflation – when output is low, whereas casual empiricism, the standard doctrine, and empirical estimates suggest the other sigh.

Most papers stop at showing that they can generate lagged inflation effects in the Phillips curve, but do not show that lagged inflation in the Phillips curve generates better monetary policy responses in a full model. This is not criticism. They do not make that claim. But a reader should not jump to that implication.

Steinsson (2003) introduces lagged inflation in the Phillips curve by positing two types of firms. "One acts rationally given Calvo-type constraints on price setting. The other type sets prices according to a rule-of-thumb. This results in a Phillips curve with both a forward-looking term and a backward-looking term." As unobjectionable as they are if they added dynamics to a model that got the basic sign right, it would again be a shame if rule-of-thumb price setters were a minimal necessary ingredient. Steinsson presents responses to supply shocks, which jump and then revert, however, not monetary policy shocks comparable to that presented in the text.

Galí and Gertler (1999) also motivate the Phillips curve with lagged and future terms by rule of thumb price setters. They evaluate this Phillips curve empirically, and despite the Ball puzzle, find "Backward-looking price setting, while statistically significant, is not quantitatively important." They conclude that "the New Keynesian Phillips curve provides a good first approximation to the dynamics of inflation." However, this is a single-equation estimation, not an evaluation of whether a full model produces the desired effects of monetary policy. Furthermore, the centerpiece of their analysis is to use marginal cost, not output, as the quantity variable. This is what the theory asks for, but it means that we resolve the Ball puzzle by driving a significant wedge between output or unemployment and marginal cost. The Phillips curve no longer tracks those standard business cycle measures.

Christiano, Eichenbaum, and Trabandt (2016, 2018) and Smets and Wouters (2007) include such backward-looking terms. Inflation still jumps in the latter with only a small hump-shape, so holding prices and wages fixed in the former is also an important ingredient. They motivate lagged inflation in the Phillips curve by price and wage indexation: Firms or workers who do not get to change price or wage this period, nonetheless raise prices or wages according to the index. However, we observe very little indexation in the US economy. If we use lagged inflation justified by indexation, then once we observe that price-setters do not index to inflation, we have to conclude that higher interest rates lose any power to lower inflation.

Choi et al. (2025) return to menu costs in an effort to produce more inflation "persistence," meaning lagged inflation in the Phillips curve. In place of the Rotemberg (1982) quadratic costs, Choi et al. post that firms pay a fixed cost proportional to their productivity and an idiosyncratic shock in order to change prices. Then firms can change prices by any amount. Choi et. al. present a numerical computation of the model in response to an exogenous AR(1) process for nominal demand, like Mankiw and Reis (2002), rather than a full model with an IS curve and interest rate shocks. They do not derive an aggregate Phillips curve. Instead, they cleverly run regressions of inflation on lagged inflation and expected future marginal costs in the simulated data from their model with the AR(1) demand shock. They find a coefficient on lagged inflation in the estimated Phillips curve. The trouble with this procedure is that we do not know if the coefficients on lagged inflation are structural, policy-invariant objects.

A.4 Sticky Information, Expectations, Higher-order Beliefs, and Rational Inattention

Sticky information and related information processing frictions have a long literature. "Rational inattention" is a related foundation for a Lucas-style Phillips curve and, deeper, for a delayed response. Much of this literature requires sophisticated modeling and techniques. But all good ideas get simpler over time.

Sims (2003) offers limited signal-processing capacity as an analytical foundation for rational attention, and an alternative to the kind of signal-extraction problem that Lucas modeled. It also naturally leads to greater and more flexible lags in incorporating aggregate information into individual decisions. One might be able to construct a rational attention foundation for what I call a generalized Lucas Phillips curve.

Woodford (2003a) revives delayed responses through information processing limits and limited attention. It begins a long series of articles by Woodford on imprecise cognition and limited rationality. Woodford's central point here is about higher-order expectations. He starts with a model of monopolistic competition, in which a firm cares about the prices others will charge and the aggregate price level. Then, higher-order expectations—expectations of the expectations of others—matter. Woodford shows that information processing costs make those expectations slow to move. He calculates the response to an exogenous disturbance to nominal GDP, or money supply. He shows a slow response of output and slower response of the price level. He does not show the interest rate, or how to merge the model with an interest-rate target. In a comment, Svensson (2003) offers a computation in which the central bank does not follow such a pure nominal GDP target, and gets quite different responses, leading him to worry that the "results are very sensitive to assumptions about monetary policy and the information used by the firms." Most of all, I echo the usual hope that we do not need higher-order expectations to find the minimal *necessary* assumptions for a model that generates the basic sign of how monetary policy affects inflation.

Maćkowiak, Matějka, and Wiederholt (2023) survey the rational inattention literature following Sims. Maćkowiak and Wiederholt (2009) apply rational inattention to sticky prices. Since idiosyncratic demand changes are so much larger than aggregate ones, "Firms then allocate almost all attention to idiosyncratic conditions. Hence, prices react strongly and quickly to idiosyncratic shocks, but only weakly and slowly to aggregate shocks. Thus, the aggregate price level responds slowly to shocks." These too are, however, illustrative numerical simulations of what happens with exogenous AR(1) demand shocks with a simple profit function, not yet a simple aggregate Phillips curve that can be united with IS and other elements of a general equilibrium model, and especially exhibited in a tractable minimalist model of the sort I pursue.

Maćkowiak and Wiederholt (2015) construct an equilibrium model in the new-Keynesian tradition, with monetary policy that follows a Taylor rule. They eschew "all sources of slow adjustment that usually are in New Keynesian models (Calvo pricing, habit formation in consumption, Calvo wage setting), instead featuring rational inattention... as the only source of slow adjustment." Alas, their model requires a nonstandard numerical solution. Like most of the empirically-oriented new-Keynesian literature, they focus on the output response. They write a conventional Taylor rule, and their response to a monetary policy shock is the same as the three equation model: a jump down followed by rise.

Even without Sims' framing in terms of Shannon capacity, the idea of rational inattention leading to slow adjustment has been pursued in a number of contexts. Near-rationality is sensible: the costs of not incorporating quickly are often small, so even small costs of information acquisition and processing can motivate slow adjustment, especially when aggregate information lives in a flood of more important idiosyncratic information. The monthly CPI is not that important when you're shopping for a cheap dinner. Reis (2006) uses rational inattention ideas to motivate slow adjustment of consumption to aggregates.

Nimark (2008) looks for a better microfoundation for a mixed Phillips curve than rule-of-thumb price setters. He introduces, in effect, higher-order expectations, "the optimal price of an individual good depends positively on a firm's own marginal cost and the price chosen by other firms, but individual firms cannot observe the marginal cost of other firms and therefore do not know the current price chosen by other firms with certainty. This set up may be referred to as firms having imperfect common knowledge in an environment with strategic complementarities." As a result "firms 'herding' on the publicly observable lagged aggregate variables, inflation and output. This creates the appearance of inflation being partly 'backward looking' in spite of the fact that all firms are rational and forward looking.

Dupor, Kitamura, and Tsuruga (2010) propose a model with both sticky prices and sticky information. The result is to add a backward-looking inflation term to a Phillips curve that otherwise resembles Mankiw and Reis'. Such backward looking terms help to fit the data. Alas, as with Mankiw and Reis, "it is generally impossible to derive the closed form solution to the dual stickiness model, due to infinitely many lagged expectations terms."

Auclert, Rognlie, and Straub (2020) is an important new paper aimed at these issues in the burgeoning HANK tradition. They aim straight at the fact that "the macroeconomic response to aggregate shocks tends to be hump-shaped, " quoting also (Woodford, 2003b, p. 322), "monetary policy can have little immediate effect on either real activity or inflation." They note that " the literature to date has used representative-agent models enhanced with a variety of adjustment frictions such as habit formation, or deviations from rational expectations such as inattention." Attention to micro MPC estimates, a central concern of HANK modeling, makes the problem worse: High MPCs lead to "an immediate jump on impact." They construct a HANK model that *also* includes "sticky expectations, sticky prices and wages with indexation, long-term debt, and investment adjustment costs." They can produce a price level that rises slowly after an AR(1)-like nominal interest rate decline. (Figure 4, bottom left panel.) A notable feature is that they trace the fiscal foundations of the model, as volume of government debt matters to the liquidity-constrained consumers of HANK models. Though they make progress on the computational difficulty of HANK models, it remains far from the sort of minimal textbook model that I search for, to replace the standard IS and Calvo Phillips curves.

Kwicklis (2025) also adds sticky expectations to a medium-scale HANK model. His model also includes "slow-adjusting Taylor and debt stabilization rules, a backward-indexed New-Keynesian

Phillips curve, and other shocks and frictions standard to Smets and Wouters (2007)." Like others, he introduces price indexing to generate lagged inflation in the Phillips curve.

Carroll et al. (2020) use the idea of sticky expectations in consumption. They want to understand why aggregate consumption seems "excessively smooth," in not behaving like a random walk while individual consumption growth is essentially unpredictable. The construct a model in which people incorporate aggregate information more slowly than individual information.

Gabaix (2020) pursues a different approach to the same questions, and for once also ends up with a tractable and simple model. "Cognitive discounting," a form of limited information-processing capacity or limited information, leads Gabaix to

$$x_t = ME_t x_{t+1} - \sigma(i_t - E_t\pi_{t+1})$$
$$\pi_t = \beta M^f E_t \pi_{t+1} + \kappa x_t$$

with $M < 1$ and $M^f < 1$.

The basic properties of Gabaix' model can be seen with $M = M^f = 0$. ($M = 0$ and $M^f > 0$ is just as easy to analyze.) Then, eliminating x_t, we have

$$E_t\pi_{t+1} = \frac{1}{\sigma\kappa}\pi_t + i_t.$$

With small intertemporal substitution and large price stickiness, $\sigma\kappa < 1$, this model displays unstable dynamics, even under a fixed interest-rate path. (My language, Gabaix labels things differently.) The model has a unique locally bounded equilibrium,

$$\pi_t = -E_t \sum_{j=0}^{\infty} (\sigma\kappa)^{j+1}\, i_{t+j}. \tag{33}$$

With an AR(1) interest-rate path,

$$\pi_t = -\frac{\sigma\kappa}{1-\sigma\kappa\rho} i_t.$$

Just like the standard model with a Taylor rule, higher interest rates make inflation jump down, and then inflation rises back to zero.

Gabaix's basic model does not, and does not try to, produce lower inflation going forward or a hump-shaped response, which is the centerpiece of the generalized Lucas Phillips curve. Related, it does not move the model towards the old-Keynesian model, with lagged terms in the

IS or Phillips curve, whether motivated by adaptive expectations or price indexing. The adaptive expectations model (replace $E_t\pi_{t+1}$ with π_{t-1} in the standard model) is also unstable, but it has no jump variable, so inflation spirals break out absent a Taylor rule. Gabaix' model is unstable, but with $E_t\pi_{t+1}$ remaining even with $M = M^f = 0$, it preserves inflation as a jump variable.

Gabaix's model solves the forward guidance puzzle in a novel way. In standard new-Keynesian models inflation is stable with a fixed interest rate; inflation converges to the interest rate. Stable forward is unstable backward, so small equilibrium-selection promises further in the future have larger effects today (See Cochrane, 2017a for this interpretation.) Since Gabaix's model, with the rule against explosive equilibria, produces unstable dynamics going forward, it is stable backward, and changes further in the future have smaller effects today, as you can see directly in (33).

With $\beta M^f < 1$, Gabaix's Phillips curve does not display long-run neutrality, which has been the center of Phillips curve thinking since Friedman (1968). Permanently higher nominal interest rates permanently lower inflation. Gabaix restores long-run neutrality by redefining the Phillips curve in terms of the difference between actual and a long-run "default" inflation. Then only the $i_t - E_t\pi_{t+1}$ of the IS curve remains describing long-run inflation, and the model restores the long-run neutrality and long-run Fisherian property that I have argued is important. The extension to default inflation also adds some old-Keynesian features, while maintaining some forward looking behavior.

García-Schmidt and Woodford (2019) investigate a "reflective equilibrium" concept, related to higher-order expectations, to avoid the Fisherian implications of the standard new-Keynesian model. They "model the economy as being in a temporary equilibrium with reflective expectations (or "reflective equilibrium" for short). By temporary equilibrium we mean,...that market outcomes at any point in time result from optimizing decisions by households and firms under expectations that are specified in the model, but that need not be correct. By reflective expectations we mean expectations formed on the basis of reasoning about how the economy should evolve, under a correct understanding of the structural relations that determine market outcomes,.." The model produces downweighting (myopia) but not lagged inflation in the Phillips curve, and thus not the hump shape. That is not their point, however.

Angeletos and Huo (2021) focus on higher-order expectations. They find that higher-order expectations can mimic myopia (less weight on future variables, as Gabaix models) and lagged inflation in the Phillips curve. As in many of these models, the connection from informational primitives to Phillips curve coefficients is quite complex, though the final answer has the sort of simplicity I have praised.

Kiley (2007) and Coibion (2010) test sticky-price vs. sticky-information Phillips curves, finding the former performs better (or less badly). Their technique is essentially regressions of inflation on measures of inflation expectations. (Kiley adds unit labor costs and Coibon adds the output gap.) These findings are not necessarily bad news for our purposes: Which view can produce a textbook-size economically based general equilibrium model consistent with the standard view of monetary policy? Regression fits of individual model equations, with necessary identification assumptions to control for endogenous right-hand variables, are not necessarily good guides to an ingredient's contribution to policy analysis in general equilibrium models.

In a detailed empirical investigation, Klenow and Willis (2007), examining micro data, come to the opposite conclusion, reporting that "Empirical price changes react to old information, just as sticky information models predict."

B Algebra

B.1 Fiscal calculations

To calculate the fiscal surplus needed to support a given inflation path, or to choose the initial price level and thus inflation path that requires a specific value of the surplus, I use the inflation solution (15) and the fiscal equation (24), which I repeat here:

$$\int_{t=0}^{\infty} e^{-rt}\tilde{s}_t = \int_{t=0}^{\infty} e^{-rt}(i_t - \pi_t)dt - p_0 - \int_{t=0}^{\infty} e^{-(r+\omega)t} i_t dt. \tag{34}$$

$$\pi_t = \frac{(1+\sigma\kappa)\, ae^{-at}}{(e^{-at}+\sigma\kappa)^2} p_0 + \frac{\sigma\kappa a e^{-at}}{(e^{-at}+\sigma\kappa)^2}\int_{j=0}^{t} i_j dj + \frac{\sigma\kappa}{e^{-at}+\sigma\kappa} i_t. \tag{35}$$

To calculate the required surplus numerically, one can simply evaluate the integrals on the right hand side of (34). To find p_0 or ω given the other variables, I substitute (35) in (34). First, write

$$i_t - \pi_t = -\frac{(1+\sigma\kappa)\, ae^{-at}}{(e^{-at}+\sigma\kappa)^2} p_0 - \frac{\sigma\kappa a e^{-at}}{(e^{-at}+\sigma\kappa)^2}\int_{j=0}^{t} i_j dj + \frac{e^{-at}}{e^{-at}+\sigma\kappa} i_t.$$

Then substitute in (34) to obtain

$$\int_{t=0}^{\infty} e^{-rt}\tilde{s}_t = -p_0\left[1+\int_{t=0}^{\infty}\frac{(1+\sigma\kappa)\,ae^{-(r+a)t}}{\left(e^{-at}+\sigma\kappa\right)^2}dt\right]$$
$$+\int_{t=0}^{\infty}\left(-\frac{\sigma\kappa a e^{-(r+a)t}}{\left(e^{-at}+\sigma\kappa\right)^2}\int_{j=0}^{t} i_j dj+\frac{e^{-(r+a)t}}{e^{-at}+\sigma\kappa}i_t\right)dt-\int_{t=0}^{\infty}e^{-(r+\omega)t}i_t dt.$$

The first term on the right hand side does not depend on the interest-rate path, and only the last term contains ω. Thus, one can add solutions, for example calculating the answer for a combination of AR(1) from the solutions for a simple AR(1). I will nonetheless present the formula for a sum of two AR(1) directly. Using

$$i_t = i_s(e^{-\eta_1 t} - we^{-\eta_2 t})$$

we have

$$\int_{t=0}^{\infty} e^{-rt}\tilde{s}_t = -p_0\left[1+\int_{t=0}^{\infty}\frac{(1+\sigma\kappa)\,ae^{-(r+a)t}}{\left(e^{-at}+\sigma\kappa\right)^2}dt\right]$$
$$-\,i_s\int_{t=0}^{\infty}\frac{\sigma\kappa a e^{-(r+a)t}}{\left(e^{-at}+\sigma\kappa\right)^2}\left(\frac{1-e^{-\eta_1 t}}{\eta_1}-w\frac{1-e^{-\eta_2 t}}{\eta_2}\right)dt$$
$$+\,i_s\int_{t=0}^{\infty}\frac{e^{-(r+a)t}}{e^{-at}+\sigma\kappa}(e^{-\eta_1 t}-we^{-\eta_2 t})dt$$
$$-\,i_s\left(\frac{1}{r+\omega+\eta_1}-w\frac{1}{r+\omega+\eta_2}\right)$$

I evaluate the integrals numerically. Setting $\tilde{s}_t = 0$, we can then solve for p_0. Alternatively, given a surplus and a value of p_0, we can solve for the ω that produces them.

B.2 Downweighted expectations

The IS is (25)

$$m\frac{dx_t}{dt} - (1-m)x_t = \sigma\left(i_t - \frac{dp_t}{dt}\right).$$

Use the Phillips curve (8) and its derivative (9)

$$\kappa x_t = \Phi(t)p_t,$$

$$\kappa\frac{dx_t}{dt} = -\alpha_t p_t + \Phi(t)\frac{dp_t}{dt}.$$

Eliminating x_t and dx/dt, the price response function solves the differential equation

$$-m\alpha_t p_t + m\Phi(t)\frac{dp_t}{dt} - (1-m)\Phi(t)p_t = \sigma\kappa\left(i_t - \frac{dp_t}{dt}\right)$$

$$[m\Phi(t) + \sigma\kappa]\frac{dp_t}{dt} - [(1-m)\Phi(t) + m\alpha_t]p_t = \sigma\kappa i_t.$$

For the exponential case,

$$\left(me^{-at} + \sigma\kappa\right)\frac{dp}{dt} - \left[1 - m(1-a)\right]e^{-at}p_t = \sigma\kappa i_t.$$

The initial condition linking inflation, price level and interest rate is

$$p_0 = \frac{m\pi_0 + \sigma\kappa(\pi_0 - i_0)}{1 - m(1-a)}$$

The integrating factor is

$$[me^{-at} + \sigma\kappa]^{\frac{1-m(1-a)}{ma}} = [me^{-at} + \sigma\kappa]^{1+\frac{1-m}{ma}}.$$

Check:

$$\frac{dp}{dt} - \frac{\left[1 - m(1-a)\right]e^{-at}}{(me^{-at} + \sigma\kappa)}p_t = \frac{\sigma\kappa}{(me^{-at} + \sigma\kappa)}i_t$$

$$[me^{-at}+\sigma\kappa]^{1+\frac{1-m}{ma}}\frac{dp}{dt} - [me^{-at}+\sigma\kappa]^{1+\frac{1-m}{ma}}\frac{\left[1 - m(1-a)\right]e^{-at}}{(me^{-at} + \sigma\kappa)}p_t = [me^{-at}+\sigma\kappa]^{1+\frac{1-m}{ma}}\frac{\sigma\kappa}{(me^{-at} + \sigma\kappa)}i_t$$

$$[me^{-at} + \sigma\kappa]^{1+\frac{1-m}{ma}}\frac{dp}{dt} - [me^{-at} + \sigma\kappa]^{\frac{1-m}{ma}}\left[1 - m(1-a)\right]e^{-at}p_t = [me^{-at} + \sigma\kappa]^{\frac{1-m}{ma}}\sigma\kappa i_t$$

$$d\left[(me^{-at} + \sigma\kappa)^{\frac{1-m(1-a)}{ma}}p_t\right] = [me^{-at} + \sigma\kappa]^{\frac{1-m}{ma}}\sigma\kappa i_t$$

The solution is

$$(me^{-at} + \sigma\kappa)^{\frac{1-m(1-a)}{ma}}p_t - (m + \sigma\kappa)^{\frac{1-m(1-a)}{ma}}p_0 = \sigma\kappa\int_{s=0}^{t}[me^{-as} + \sigma\kappa]^{\frac{1-m}{ma}}i_s ds$$

$$p_t = \left(\frac{m + \sigma\kappa}{me^{-at} + \sigma\kappa}\right)^{\frac{1-m(1-a)}{am}}p_0 + \frac{\sigma\kappa}{me^{-at} + \sigma\kappa}\int_{s=0}^{t}\left(\frac{me^{-as} + \sigma\kappa}{me^{-at} + \sigma\kappa}\right)^{\frac{1-m}{am}}i_s ds.$$

Inflation follows

$$\pi_t = e^{-at}\left(\frac{m + \sigma\kappa}{me^{-at} + \sigma\kappa}\right)^{\frac{1-m}{am}+2}\frac{m\pi_0 + \sigma\kappa(\pi_0 - i_0)}{m + \sigma\kappa} +$$

$$+[1-m(1-a)]\frac{\sigma\kappa e^{-at}}{(me^{-at}+\sigma\kappa)^2}\int_{s=0}^{t}\left(\frac{me^{-as}+\sigma\kappa}{me^{-at}+\sigma\kappa}\right)^{\frac{1-m}{am}} i_s ds$$
$$+\frac{\sigma\kappa}{me^{-at}+\sigma\kappa}i_t.$$

B.3 Habits and Calvo Pricing

Define

$$dX_t = b(x_t - X_t)dt$$

The first order condition is

$$E_t dx_t - \theta b(x_t - X_t)dt = \sigma[i_t dt - E_t(dp_t)].$$

The impulse response is

$$\frac{dX}{dt} = bx_t - bX_t$$
$$\frac{dx_t}{dt} - \theta b(x_t - X_t) = \sigma(i_t - \pi_t)$$
$$\frac{d\pi_t}{dt} = r\pi_t - \kappa x_t.$$

Write it in matrix form

$$\frac{d}{dt}\begin{bmatrix} X_t \\ x_t \\ \pi_t \\ i_t \end{bmatrix} = \begin{bmatrix} -b & b & 0 & 0 \\ -\theta b & \theta b & -\sigma & \sigma \\ 0 & -\kappa & r & 0 \\ 0 & 0 & 0 & -\eta_i \end{bmatrix}\begin{bmatrix} X_t \\ x_t \\ \pi_t \\ i_t \end{bmatrix}$$

There is one forward-looking (positive real part) eigenvalue and two jump variables, π and x. I specify π_0 and use the explosive root to find x_0.

B.4 Lags in the Phillips Curve

The model is

$$x_t = E_t x_{t+1} - \sigma(i_t - E_t\pi_{t+1})$$
$$\pi_t = (1-\gamma)E_t\pi_{t+1} + \gamma\pi_{t-1} + \kappa x_t$$

One may solve by the usual matrix method, writing

$$
\begin{aligned}
(1-\gamma)E_t\pi_{t+1} &= \pi_t - \gamma\pi_{t-1} - \kappa x_t \\
E_t x_{t+1} &= x_t + \sigma i_t - \frac{\sigma}{1-\gamma}\left(\pi_t - \gamma\pi_{t-1} - \kappa x_t\right) \\
E_t x_{t+1} &= \left(1 + \frac{\sigma\kappa}{1-\gamma}\right) x_t + \sigma i_t - \frac{\sigma}{1-\gamma}\left(\pi_t - \gamma\pi_{t-1}\right)
\end{aligned}
$$

I pursue a lag operator solution, which allows an analytic solution, up to finding the roots of the lag polynomial. Using the Phillips curve at time $t+1$, we can eliminate x_t,

$$
E_t\pi_{t+1} - (1-\gamma)E_t\pi_{t+2} - \gamma\pi_t - \pi_t + (1-\gamma)E_t\pi_{t+1} + \gamma\pi_{t-1} = \sigma\kappa(i_t - E_t\pi_{t+1})
$$

$$
-(1-\gamma)E_t\pi_{t+2} + (1 + (1-\gamma) + \sigma\kappa)E_t\pi_{t+1} - (1+\gamma)\pi_t + \gamma\pi_{t-1} = \sigma\kappa i_t
$$

$$
\left[-\frac{1-\gamma}{\sigma\kappa}L^{-1} + \left(\frac{2-\gamma}{\sigma\kappa} + 1\right) - \frac{1+\gamma}{\sigma\kappa}L + \frac{\gamma}{\sigma\kappa}L^2\right]\pi_{t+1} = i_t.
$$

Note

$$
a(1) = \left[-\frac{1-\gamma}{\sigma\kappa} + \left(\frac{2-\gamma}{\sigma\kappa} + 1\right) - \frac{1+\gamma}{\sigma\kappa} + \frac{\gamma}{\sigma\kappa}\right] = 1.
$$

Thus, we want

$$
\frac{(1-\lambda_1 L^{-1})(1-\lambda_2 L)(1-\lambda_3 L)}{(1-\lambda_1)(1-\lambda_2)(1-\lambda_3)}\pi_{t+1} = i_t. \tag{36}
$$

The Matlab root-finding program wants polynomials with non-negative powers, so transform to

$$
\left[-\frac{1-\gamma}{\sigma\kappa} + \left(\frac{2-\gamma}{\sigma\kappa} + 1\right)L - \frac{1+\gamma}{\sigma\kappa}L^2 + \frac{\gamma}{\sigma\kappa}L^3\right]L^{-1}\pi_{t+1} = i_t
$$

$$
\frac{-(1-\lambda_1^{-1}L)(1-\lambda_2 L)(1-\lambda_3 L)\lambda_1 L^{-1}}{(1-\lambda_1)(1-\lambda_2)(1-\lambda_3)}.
$$

I find the roots of the polynomial in square brackets, and these are then λ_1, λ_2^{-1} and λ_3^{-1}.

Then, we invert (36)

$$
\pi_{t+1} = \frac{(1-\lambda_1)(1-\lambda_2)(1-\lambda_3)}{(1-\lambda_1 L^{-1})(1-\lambda_2 L)(1-\lambda_3 L)} i_t.
$$

I expand the right-hand side by partial fractions, in steps. First,

$$\frac{1}{(1-\lambda_2 L)(1-\lambda_3 L)} = \frac{1}{\lambda_2-\lambda_3}\left(\frac{\lambda_2}{1-\lambda_2 L} - \frac{\lambda_3}{1-\lambda_3 L}\right).$$

Then,

$$\frac{1}{(1-\lambda_1 L^{-1})(1-\lambda_2 L)} = \frac{1}{(1-\lambda_1\lambda_2)}\left(\frac{1}{1-\lambda_1 L^{-1}} + \frac{\lambda_2 L}{1-\lambda_2 L}\right),$$

so

$$\frac{1}{(1-\lambda_1 L^{-1})(1-\lambda_2 L)(1-\lambda_3 L)} = \frac{1}{\lambda_2-\lambda_3}\frac{1}{(1-\lambda_1 L^{-1})}\left(\frac{\lambda_2}{1-\lambda_2 L} - \frac{\lambda_3}{1-\lambda_3 L}\right)$$

$$= \frac{1}{\lambda_2-\lambda_3}\left[\frac{\lambda_2}{(1-\lambda_1\lambda_2)}\left(\frac{1}{1-\lambda_1 L^{-1}} + \frac{\lambda_2 L}{1-\lambda_2 L}\right) - \frac{\lambda_3}{(1-\lambda_1\lambda_3)}\left(\frac{1}{1-\lambda_1 L^{-1}} + \frac{\lambda_3 L}{1-\lambda_3 L}\right)\right].$$

Including the free constant, then

$$\pi_{t+1} = C\lambda_1^{t+1} + \frac{(1-\lambda_1)(1-\lambda_2)(1-\lambda_3)}{\lambda_2-\lambda_3}\times$$
$$\left[\frac{\lambda_2}{(1-\lambda_1\lambda_2)}\left(\frac{1}{1-\lambda_1 L^{-1}} + \frac{\lambda_2 L}{1-\lambda_2 L}\right) - \frac{\lambda_3}{(1-\lambda_1\lambda_3)}\left(\frac{1}{1-\lambda_1 L^{-1}} + \frac{\lambda_3 L}{1-\lambda_3 L}\right)\right] i_t.$$

Writing out the lag operators,

$$\pi_{t+1} = C\lambda_1^{t+1} + \frac{(1-\lambda_1)(1-\lambda_2)(1-\lambda_3)}{\lambda_2-\lambda_3}\times$$
$$\left[\frac{\lambda_2}{1-\lambda_1\lambda_2}\left(\sum_{j=0}^{\infty}\lambda_1^j i_{t+j} + \sum_{j=1}^{\infty}\lambda_2^j i_{t-j}\right) - \frac{\lambda_3}{1-\lambda_1\lambda_3}\left(\sum_{j=0}^{\infty}\lambda_1^j i_{t+j} + \sum_{j=1}^{\infty}\lambda_3^j i_{t-j}\right)\right].$$

For an AR(1) interest rate $i_t = i_0\eta_i^t$, $i_t = 0,\ t<0$,

$$\pi_{t+1} = C\lambda_1^{t+1} + i_0\frac{(1-\lambda_1)(1-\lambda_2)(1-\lambda_3)}{\lambda_2-\lambda_3}\times$$
$$\left[\frac{\lambda_2}{(1-\lambda_1\lambda_2)}\left(\sum_{j=0}^{\infty}\lambda_1^j\eta_i^{t+j} + \sum_{j=1}^{t}\lambda_2^j\eta_i^{t-j}\right) - \frac{\lambda_3}{(1-\lambda_1\lambda_3)}\left(\sum_{j=0}^{\infty}\lambda_1^j\eta_i^{t+j} + \sum_{j=1}^{t}\lambda_3^j\eta_i^{t-j}\right)\right].$$

Note

$$\sum_{j=1}^{t}\lambda_2^j\eta_i^{t-j} = \frac{\eta_i^{t-1}\lambda_2(1-\lambda_2^t\eta_i^{-t})}{1-\lambda_2\eta_i^{-1}} = \frac{\lambda_2(\eta_i^t-\lambda_2^t)}{\eta_i-\lambda_2}.$$

Therefore

$$\pi_{t+1} = C\lambda_1^{t+1} + i_0\frac{(1-\lambda_1)(1-\lambda_2)(1-\lambda_3)}{\lambda_2-\lambda_3}\times$$

$$\left[\frac{\lambda_2}{(1-\lambda_1\lambda_2)}\left(\frac{\eta_i^t}{1-\lambda_1\eta_i}+\frac{\lambda_2(\eta_i^t-\lambda_2^t)}{\eta_i-\lambda_2}\right)-\frac{\lambda_3}{(1-\lambda_1\lambda_3)}\left(\frac{\eta_i^t}{1-\lambda_1\eta_i}+\frac{\lambda_3(\eta_i^t-\lambda_3^t)}{\eta_i-\lambda_3}\right)\right]$$

and finally

$$\pi_{t+1}=C\lambda_1^{t+1}+i_0\frac{(1-\lambda_1)(1-\lambda_2)(1-\lambda_3)}{\lambda_2-\lambda_3}\times$$
$$\left[\frac{\lambda_2}{\eta_i-\lambda_2}\left(\frac{\eta_i^{t+1}}{1-\lambda_1\eta_i}-\frac{\lambda_2^{t+1}}{1-\lambda_1\lambda_2}\right)-\frac{\lambda_3}{\eta_i-\lambda_3}\left(\frac{\eta_i^{t+1}}{1-\lambda_1\eta_i}-\frac{\lambda_3^{t+1}}{1-\lambda_1\lambda_3}\right)\right].$$