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Does Government Debt Management Matter? High-Frequency Identification From U.S. Treasury Quarterly Refunding Announcements

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This paper measures the impact of surprises in the aggregate and relative supply of U.S. Treasury securities at different maturities on Treasury yields using a new measure of supply surprises. Surprises are measured as the difference between the supply announced by the U.S. Treasury Department at Quarterly Refunding Announcements and a novel dataset of pre-announcement expectations of supply from primary dealers. The reaction of Treasury yields is measured in a tight intraday window around the announcement, to precisely identify the causal effect of supply surprises on yields. We find that the unexpected supply of one percent point of GDP of ten-year-equivalent face value of debt in the next 3 months is associated with 0.07% to 0.20% increase in the 10-year minus 3-month term spread. This is larger than other estimates in the literature. The existence of supply effects is consistent with a simple theory of term premia adjusting to the total quantity of duration supply: imperfect risk sharing in an overlapping-generations macroeconomic model is sufficient to break Ricardian equivalence. Using maturity-specific issuance surprises, we show that supply shocks at short and intermediate maturities transmit strongly to longer yields, while long-end issuance has no independent effect on intermediate rates, a pattern that supports an additional role for preferred-habitat theories of segmented demand and imperfect arbitrage. We derive implications of these empirical effects in a version of a model of optimal debt management used by the U.S. Treasury Borrowing Advisory Committee. We find that recalibrating the magnitude of Treasury supply effects in line with our estimates lowers the optimal weighted-average maturity in favor of shorter-duration government debt issuance.

Keywords: Government Debt Management, Debt, Deficits, Bond Interest Rates, Asset Pricing
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1 Introduction

"A tenet of Treasury debt management is that issuance should be regular and predictable." - Janet Yellen (2024)[1]

In recent years, global government debt levels have climbed to near post-World War II highs, with the average OECD general government debt reaching 110.5 percent of GDP in 2023, while U.S. federal debt held by the public rose to 98 percent of GDP as of the fourth quarter of 2025. Amidst rising sovereign debt burdens in advanced economies, debt maturity structure is an increasingly important policy choice. In this context, many policymakers and economists believe that shortening the maturity of debt issuance can help reduce the fiscal burden[2].

In this paper, we provide new quantitative estimates of effects of Treasury supply on Treasury yields, and derive implications for the policy trade-off between longer and shorter maturity issuance at the core of debt management. We find that the aggregate supply of duration risk embedded in longer-term securities raises long-term yields more than previously estimated, and using these estimates to recalibrate an extended model of optimal debt management justifies a larger role for short-term issuance. Moreover, we find that issuance shocks at different maturities have distinctive pattern of transmission to yields at various points in the term structure, in a way that cannot just be captured by the magnitude of aggregate duration supply, but supports the role of more complex theories of segmented demand and intermediation frictions, such as Vayanos and Vila (2021).

Before introducing more detail on our empirical strategy and debt management implications, here is how our findings fit in economic theory. Under a frictionless benchmark (Barro (1974)), debt management is irrelevant.[3] The lack of effect on interest rates and welfare is known as 'Ricardian Equivalence'. We present a minimal model to break this irrelevance: in an overlapping generations (OLG) model, investors have limited horizons, and must re-sell longer-term securities before maturity at a price determined by the future level of the equilibrium short rate, in turn determined by the future strength of the economy. Ruling out the existence of insurance against this aggregate risk is sufficient to allow for changes in the relative supply of long-term versus short-term government debt to cause changes in yields and term spreads. More complex models such as the aforementioned Vayanos and Vila (2021) include financial intermediaries who demand compensation for exposing their capital to the risk of inventories of Treasuries. Their role is to smooth the gaps between the segmented demand from many cohorts of OLG investors with strong preferences for specific investment horizons ("preferred habitats") and the available supply from the government, providing some of the insurance absent in the simple model, for a price that reacts to the changes in maturity-specific supply.

The empirical contribution of the paper is a novel measure of supply shocks to estimate these aggregate and maturity-specific supply effects, with the following identification strategy. We measure the high-frequency reaction of Treasury yields to surprises in U.S. Treasury Quarterly Refunding Announcements (QRAs). The surprises are defined as the difference between the announced Treasury plans for the maturity composition of debt issuance over the following 3 months and the expectations of primary dealers precisely for such announcement. We measure both an aggregate surpise and maturity-specific surprises. [4]

[1] Lawder, David. "Yellen says bill issuance not aimed at 'sugar high'", *Reuters*, October 25, 2024

[2] Duguid, Kate. "US Treasury to sell more short-term debt in continuation of Biden-era policy", *Financial Times*, July 30, 2025

[3] Different debt issuance policies by the government imply different paths of net taxation, but this in turn incentivizes infinitely-lived dynasties of households to absorb the changed supply profile at the same pricing, since this is exactly what they need to offset the change in taxation paths and restore the preferred state-contingent menu of consumption streams.

[4] QRAs are announced by the U.S. Treasury Assistant Secretary for Financial Markets. In 2024, the position was held by Joshua Frost, whom the *Wall Street Journal* called "The Most Important Man in Finance You've Never Heard Of" *The Wall*

The data on the primary dealers' expectations of Treasury issuance prior to QRAs is a novel contribution of this paper and it is crucial to estimate surprises in issuance quantities. It was hand-collected from decades of primary dealer analyst reports that forecast QRA issuance amounts by maturity in the days just prior to the QRA.

Treasury QRA surprises represent an effective way to estimate the slope of the demand curve for Treasury securities, by providing an exogenous shock in supply that is arguably not correlated with unobservables affecting demand an effective source of information on supply shocks. They have the added benefit of a breakdown by maturity that is not available at the level of U.S. Treasury funding estimates (Wachtel and Young (1987)) and CBO announcements (Gomez Cram et al. (2025)), and is therefore useful to explore difference in elasticities at different maturities.

We find that an unexpected announcement of an issuance within the next 3 months with the face value of 1% of GDP in ten-year equivalent debt securities causes a 0.07% to 0.20% increase in the 3-month to 10-year term spread. In magnitudes, these results suggest that the term spread is more sensitive to the duration of issuance than past estimates in the literature (Greenwood and Vayanos (2014), Belton et al. (2018), Wright (2022)). Moreover, across maturities, we find that shocks to shorter and intermediate maturities spill over strongly to longer yields, while long-end issuance does not independently affect intermediate rates—evidence of cross-maturity transmission and segmented demand consistent with preferred-habitat theories such as Vayanos and Vila (2021). Importantly, long-end issuance has no independent effect on intermediate yields once shorter-maturity supply is controlled for, rejecting the sufficiency if a aggregate-duration model in favor of preferred-habitat theories with segmented demand.

To derive debt management implications, we use our estimates of aggregate duration supply effects per point of GDP of ten-year equivalent face value to re-calibrate the model that the Treasury Borrowing Advisory Committee has been using to advise the Treasury Department on debt management policy (Belton et al. (2018)). Our higher estimates of supply effects twist the efficient frontier describing the tradeoff between interest cost and volatility in favor of a shorter optimal weighted-average maturity of government debt across a wide spectrum of levels of risk aversion.

In conclusion, these findings provide a useful empirical foundation for more cost-effective debt management. They suggest that a shift toward shorter-term issuance reduces interest expenses more than previously estimated, an insight relevant to the U.S. Treasury and other debt managers, especially in a context high debt-to-GDP ratios and elevated real rates.

Literature Review

Optimal government debt management, that is how to issue sovereign debt so as to minimize average interest expense subject to a level of funding-cost volatility, has been a classic question in macroeconomics for decades.[5]

In a frictionless Ricardian environment of lump-sum taxes with a Ramsey planner (Barro (1974)), debt management is irrelevant. In the presence of distortionary taxes, Barro (1979), Barro (1997) and Angeletos (2002) show that issuing longer-term bonds helps smooth the government's fiscal burden over time by allowing for gradual tax adjustments, particularly if long-term yields are stable and fiscal shocks are persistent.

However, once frictions are introduced, this tax-smoothing logic favoring long-term debt may no longer hold. Blanchard and Missale (1994) demonstrate that commitment problems and moral hazard act as countervailing forces because the government's incentive to inflate or default induces a preference for shorter-term maturities.

Similarly, incomplete markets distort the optimal maturity structure. The degree to which the available set of debt instruments spans relevant fiscal risks determines the extent to which debt maturity can serve as a tool for intertemporal risk sharing (Angeletos (2002); Lustig et al. (2008)).

Our paper also builds on the tradition of overlapping-generations (OLG) in government debt management theory, including (Barro, 1974), Bohn (1990), (Angeletos, 2002), and (Faraglia et al., 2019). These frameworks use finite-lived agents to study how government debt affects intertemporal allocations, but they differ in focus. (Barro, 1974) and

Street Journal, Jan. 27, 2024

[5]See Modigliani and Sutch (1967), Barro (1979), Barro (1997), Lucas and Stokey (1983), Bohn (1988), Bohn (1990), Campbell (1995), Angeletos (2002), Aiyagari et al. (2002), Nosbusch (2008), Lustig et al. (2008), Berndt et al. (2012), Guibaud et al. (2013), Angeletos et al. (2013), Greenwood et al. (2015), Bhandari et al. (2017a), Bhandari et al. (2019), Faraglia et al. (2019)

Bohn (1990) emphasize Ricardian equivalence and tax-smoothing under exogenous interest rates. (Angeletos, 2002) demonstrates how complete markets can replicate state-contingent debt using long maturities. (Faraglia et al., 2019) highlight institutional frictions such as no-buyback constraints. Our contribution is to repurpose the OLG structure to show that even without such institutional restrictions, incomplete intergenerational risk-sharing alone can generate equilibrium duration premia consistent with the empirical Treasury maturity shocks documented in this paper.

A further departure from the frictionless benchmark arises from the moneyness of short-term debt. A preferred-habitat theory of interest rates, modeled by Culbertson (1957), Modigliani and Sutch (1966), and Vayanos and Vila (2021),[6] suggests that heterogeneous investor demand across maturities implies that an optimal debt management strategy accounting for these "habitats" can generate fiscal savings. In a related but simpler framework, Greenwood et al. (2015) show that excess demand for money-like assets such as Treasury bills can make additional short-term issuance—relative to the existing short-term share of debt—reduce average government interest expenses.

While debt management research has usually taken a theoretical approach, empirical analyses often use time-series econometrics (Greenwood and Vayanos (2014)). Few researchers, like us, use reduced-form analysis with high-frequency identification. Of the few papers that have studied Treasury QRAs, all of which are recent, ours is the first to combine an analysis of the high-frequency effects of issuance-size announcements on Treasury yields together with data on forecasted Treasury issuance (quantities).

While many papers have studied government auctions in detail (Lou et al. (2013), Allen et al. (2023), Ray et al. (2024)), few have used QRAs for high-frequency intraday identification of the effects of government debt management across the entire U.S. government securities yield curve. Our use of QRA data and prior conditional expected issuance generates, arguably, the cleanest measure of exogenous shocks to bond supply.

Wachtel and Young (1987) study Treasury supply announcements using daily Treasury yield data, finding no effect on Treasury yields. More recently, Phillot (2021) analyzes the effects of macro announcements (including QRAs) on Treasury futures prices, without using measures of issuance expectations. Greenwood et al. (2014) using Treasury yield data at a daily frequency, find that QRA days account for a significant fraction of yield changes. Jones (2023) uses Wrightson ICAP Treasury bill expectations data (although not in high-frequency windows around QRAs) to estimate surprises in demand for short-term Treasury bills, using a local-projections instrumental variables approach with weekly data. Haddad et al. (2024) analyze U.S. Treasury Monday funding estimate announcements (FEAs) and Wednesday QRAs, using daily data and an event window from Friday close to Thursday close in weeks with a refunding announcement. Wang and Zhao (2024) study Treasury pre-refunding announcement effects, finding statistically significant drops in yields ahead of the Monday funding estimate announcements but no average yield changes either during funding estimate announcements or during Wednesday QRAs. This is consistent with our findings, which rely on duration issuance surprises (rather than purely high-frequency yield data) and which find that most QRAs are anticipated, and that the yield effects of the few positive duration surprises are offset by the yield effects of negative duration surprises.

In contrast to prior work, we estimate relative Treasury supply effects using intraday minutely Treasury yield data (within high-frequency windows around QRAs) together with high-frequency duration (issuance) surprises estimated from announced issuance data and expected issuance data from analyst forecasts.

Other related exogenous shocks involving Treasury supply effects include the 2000-2001 Treasury's Debt Buyback Program (Garbade and Rutherford (2007), Han et al. (2007)), which was a buyback program announced by Treasury in the early 2000s in response to the U.S. government's fiscal surpluses. Fleming (2002) looks at the reopenings of 52-week bills in the late 1990s as a natural experiment, finding a downward-sloping demand curve, whose cost to the government swamp the liquidity benefits.

Several papers analyze and estimate Treasury demand using data on debt supply and deficits. Krishnamurthy and Vissing-Jorgensen (2012) compare Treasury bond yields to AAA-corporate bond yields (which they argue to be similar in credit risk) over the past century to estimate how this yield spread varies with government debt-to-GDP. Wachtel

[6] Vayanos and Vila (2021) update Modigliani and Sutch (1966) by providing a formal and dynamic general-equilibrium framework for preferred-habitat theory that incorporates risk-averse arbitrageurs constrained by their risk tolerance, thereby introducing a mechanism for yield-curve smoothing and transmission of shocks across maturities.

and Young (1990) analyze CBO and OMB announcements that contain new information about deficits. Using daily Treasury yield data, they find a positive relationship between unanticipated announcements of the projected federal government deficit and interest rates.

A complementary strand of work examines how aggregate government debt levels influence asset prices more broadly. Gomez Cram et al. (2025) shows that shifts in Treasury market net supply affect the pricing of aggregate risk premia through intermediaries' balance-sheet constraints using CBO announcements about major fiscal legislation for identification, while Wiegand (2025) identifies the impact of fiscal-policy-induced debt shocks in the budget reconciliation process to measure the effects of Treasury supply on interest rates. These findings underscore that both the aggregate level and the composition of government debt can transmit to asset-price dynamics. Our results are different in that we analyze relative changes in the composition of government debt rather than changes in overall government debt. We are the first to compare shocks to the issuance size of debt in funding announcements to simultaneous shocks to Treasury yields.

Other studies attempting to measure the demand for Treasuries include those conducting event studies of quantitative easing (Krishnamurthy and Vissing-Jorgensen (2011), D'Amico and King (2013), Breedon and Turner (2016), Song and Zhu (2018), Rebucci et al. (2020), Selgrad (2024)). Those studies have limited ability to tease out the demand elasticity for Treasuries, as they combine the effects of central bank long-term asset purchase announcements, which include signals about the path of short-term interest rates, and the effects of the portfolio rebalancing channel (Krishnamurthy and Vissing-Jorgensen (2015), Selgrad (2024)).

Another approach, closer to ours, uses auction shocks to estimate the demand for government debt. For instance, Allen et al. (2023) estimate a demand system for government bonds using Canadian auction data. Boneva et al. (2020) similarly analyze UK gilt auction data. However, this approach to measuring government debt demand may suffer from measuring a combination of the market learning about Treasury demand (an information shock) in addition to a supply shock. Dos Santos (2024) exploits variation in tenor-specific government bond supplies in Brazil to identify the effect of government bond issuance on corporate bond issuance.

The rest of the paper is structured as follows. Section 2 describes the institutional background of QRAs. Section 3 documents the data and empirical strategy. Section 4 describes the high-frequency reduced-form results. Section 5 presents the results of a model that was originally built for the Treasury Borrowing Advisory Committee (substituting our new reduced-form estimates) finding that more lower maturity debt from current levels can generate fiscal savings. The model is based on a mean-variance debt manager seeking to minimize interest costs subject to a specified level of volatility. Section 6 presents an overlapping generations (OLG) model of government debt management with imperfect risk sharing to illustrate maturity policy and term premia dynamics. Section 7 concludes.

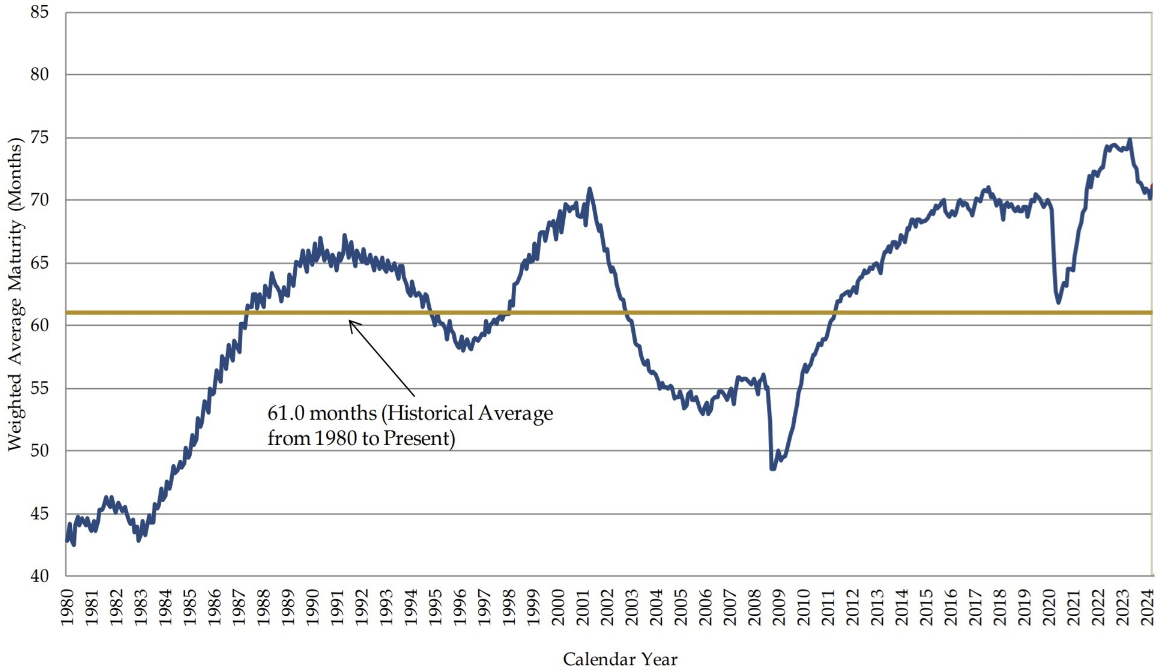
2 Institutional Details

Through its issuance maturity choices and buybacks, the U.S. Treasury can adjust the weighted-average maturity and duration of outstanding debt. This practice is commonly known as debt management policy. Since 1980, the weighted-average maturity (WAM) of Treasury debt has fluctuated between 3.5 years (43 months) in 1980 and just over 6 years (75 months) in 2023 (Figure 2). Over this period, the average WAM was 5.1 years (61 months). As of April 30, 2024, WAM stood at 5.9 years (71.1 months). Historically, the Treasury's debt management policy has led WAM to vary with the business cycle—shortening during recessions as issuance shifted toward bills to meet financing needs, and lengthening in expansions as issuance moved toward longer maturities.

In the 1970s, the Treasury began moving away from choosing note and bond maturities on an ad hoc, offering-by-offering basis. By 1982, it had adopted a system of issuing securities on a 'regular and predictable issuance.' This reduced uncertainty around Treasury announcements, facilitated investor planning, and was intended to lower borrowing costs and their volatility.[7]

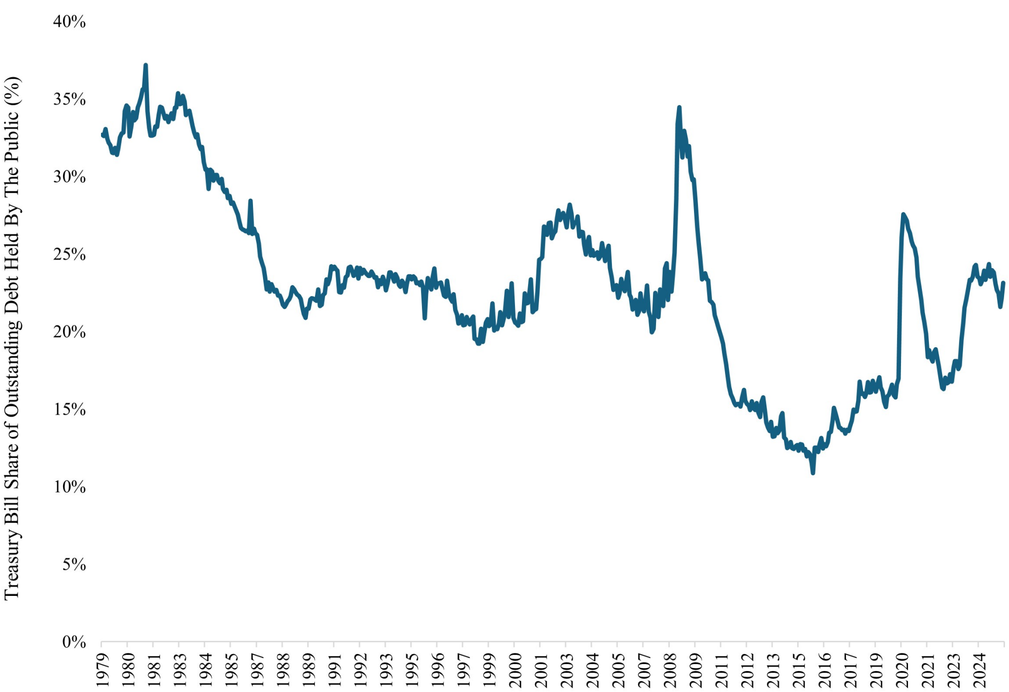
[7]Relatedly, Garbade (2007) discusses the emergence of the U.S. Treasury's "regular and predictable" issuance strategy and shift away from the prior "tactical" issuance strategy, which was conducted on an offering-by-offering basis, usually after surveying

Figure 1: Weighted-Average Maturity (WAM) of U.S. Treasury Marketable Debt Outstanding



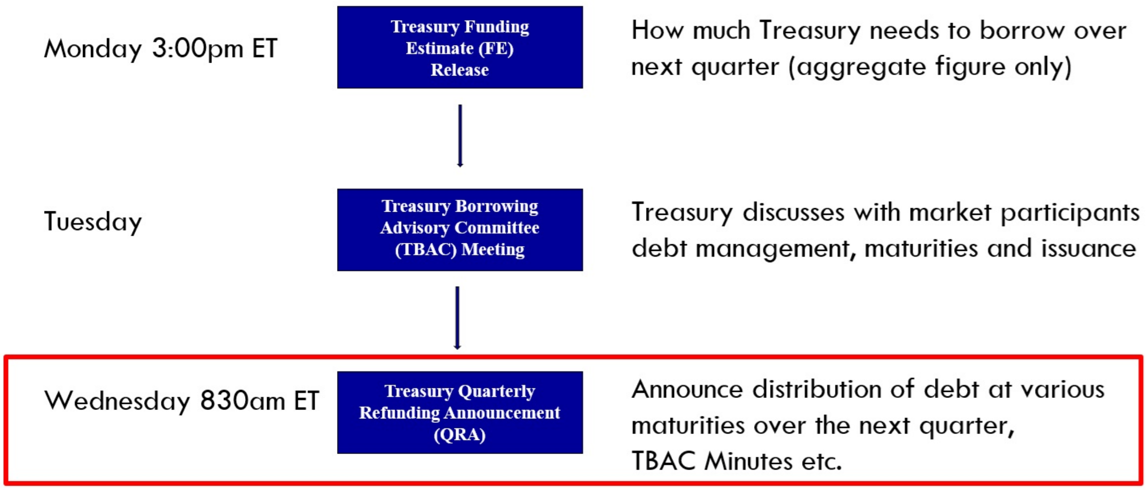
Note: This figure plots the historical weighted-average maturity (WAM) of U.S. Treasury marketable debt from 1980 to 2024. WAM has fluctuated between approximately 3.5 and 6.5 years, shortening during recessions and lengthening during expansions. Source: Treasury Borrowing Advisory Committee (TBAC).

Figure 2: U.S. Treasury Bill Share (Maturity of One Year or Less) of U.S. Treasury Marketable Debt Outstanding (%)



Note: The figure shows the share of Treasury marketable debt maturing within one year ("bills") as a percentage of total debt outstanding. The bill share rises during recessions, reflecting short-term financing needs, and declines during expansions as the Treasury extends maturities. Source: U.S. Treasury, Monthly Statement of the Public Debt.

Figure 3: U.S. Treasury Quarterly Refunding Process



Note: This figure summarizes the Treasury's quarterly refunding cycle, showing the sequence from the Quarterly Funding Estimate to the Quarterly Refunding Announcement (QRA), subsequent TBAC consultation, and detailed auction announcements. The QRA defines the maturity composition of issuance over the next quarter and provides the key policy event for this paper's empirical analysis.

As part of this process, Treasury introduced Quarterly Refunding Announcements (QRAs), which provide a high-level plan of issuance policy across the term structure over the next three months. More granular auction details are announced throughout the quarter, typically only days or weeks before auction dates. While Treasury is not legally bound to follow the quarterly issuance plan, we found that deviations are rare.

The refunding cycle begins with Treasury's Quarterly Funding Estimate, released two days before the QRA, which projects total financing needs. The QRA itself follows consultations with the Treasury Borrowing Advisory Committee (TBAC)—a group of market participants that advises Treasury on how issuance choices affect financial markets and how best to achieve the goal of least-cost financing over time.

Much like FOMC announcements, QRAs are released at a fixed, precisely scheduled time (recently 8:30 a.m. ET on the Wednesday of the funding-estimate week, typically the last week of January, April, July, and October).

In our analysis, we exploit the information content of QRAs—specifically the maturity composition of a fixed amount of debt already announced in the Funding Estimate—as a source of exogenous variation in relative bond supply at various maturities to identify how debt management policy affects government borrowing costs.

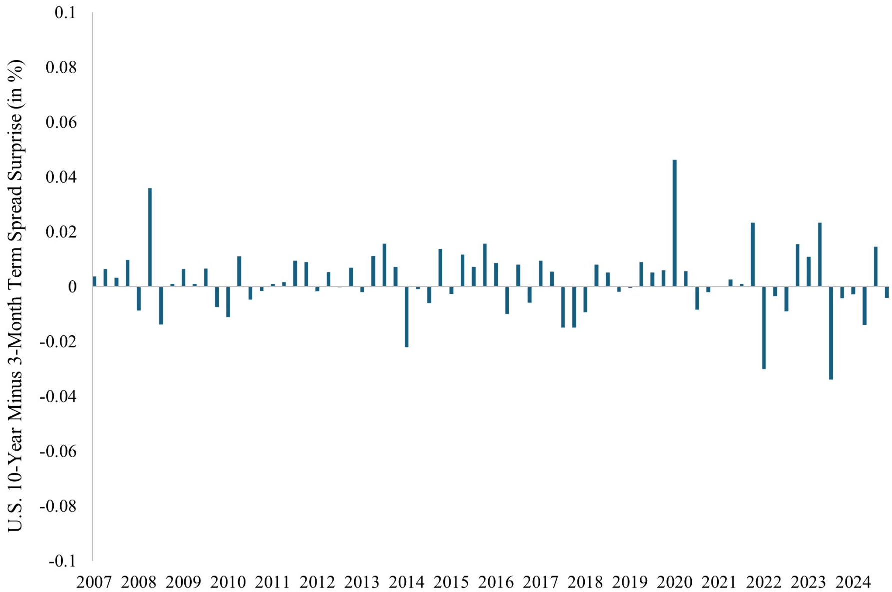
3 Data

3.1 Intraday Treasury yield data

We use minutely intraday Treasury yield data for all nominal and real U.S. Treasury maturities from 2003 to 2024, obtained from Bloomberg. To measure market reactions to Treasury supply shocks, we construct a high-frequency event window around each Quarterly Refunding Announcement (QRA), which is released at 8:30 a.m. ET (time of the U.S. Treasury's quarterly refunding announcements changed from 9:00 a.m. to 8:30 a.m. ET starting with the announcement released on Wednesday, July 31, 2013). Following the literature, we define the yield response as the difference between (i) the average yield over the 10-minute window beginning 10 minutes after the announcement and (ii) the average yield over the 10-minute window beginning 10 minutes before the announcement. Thus, our measure of the surprise in yields is given by:

market participants to identify investor demand for different maturities.

Figure 4: U.S. Treasury 3-month to 10-year Term Spread Surprises (in %) During Quarterly Refunding Announcement high-frequency Windows Over Time



Note: This figure plots the intraday change in the 3-month to 10-year Treasury term spread within a high-frequency window around each Quarterly Refunding Announcement (QRA). Each bar corresponds to a QRA event. Positive values indicate steeper yield curves following unexpected increases in long-term issuance. Source: Bloomberg.

$$\Delta r_T = \frac{1}{10}\sum_{t=1}^{10} r_{T+10+t} - \frac{1}{10}\sum_{t=1}^{10} r_{T-10+t} \tag{1}$$

Figure 4, Figure 5, and Figure 6 plot the high-frequency changes in yields for each quarter over time for the 3-month to 10-year term spread, the 10-year yield, and the 3-month yield, respectively.

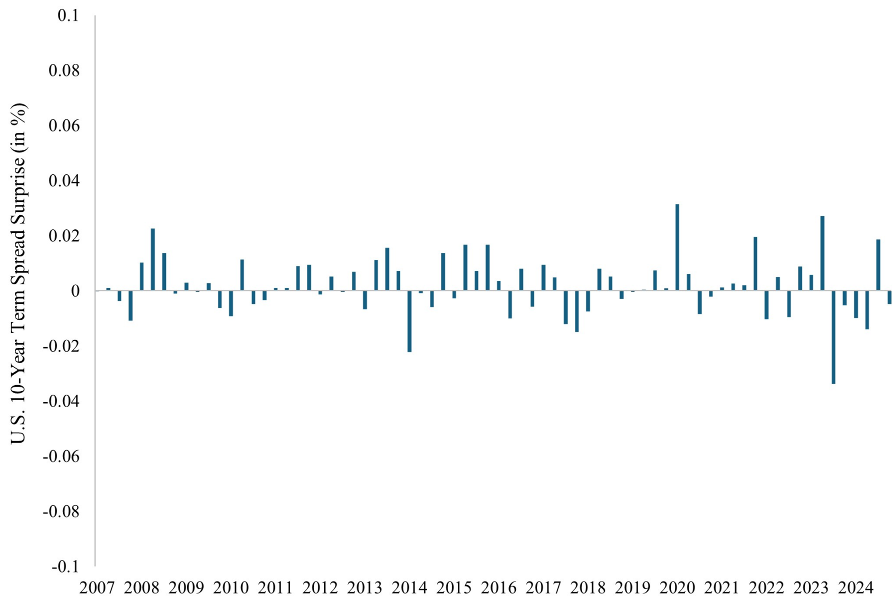
3.2 Treasury Quarterly Refunding Announcement Issuance Policy Data

Since the February 2020 Treasury Quarterly Refunding Announcement (QRA), the U.S. Treasury has included tables in the statement delivered by the Assistant Secretary for Financial Markets which include planned issuance sizes over the next quarter. For example, for the Fourth Quarter 2023 QRA on November 1, 2023, Figure A2 shows the data table release as part of the statement. The table reports issuance amounts, in billions of dollars, for the August–October 2023 quarter, as well as the Treasury's anticipated auction sizes for the upcoming November 2023–January 2024 quarter.

Before 2020, Treasury provided advance auction size announcements only for the 3-year note, 10-year note, and 30-year bond, only one month ahead, offering a relatively incomplete forecast of issuance for the upcoming quarter.

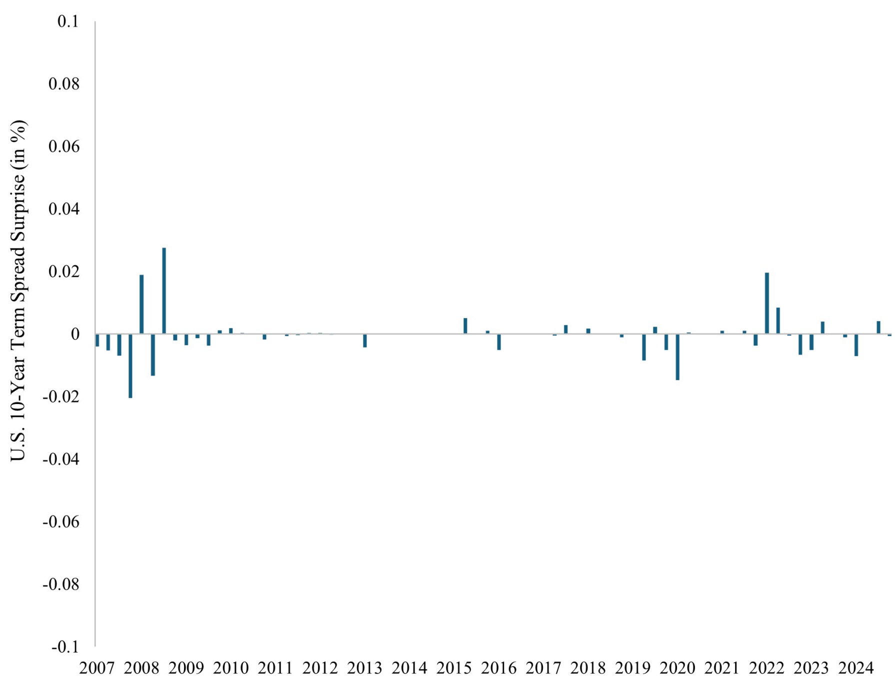
At each meeting held the day before the Quarterly Refunding Announcement, the TBAC publishes its own recommended financing tables for the next two quarters. These recommendations are often closely aligned with primary dealer issuance expectations. It is also possible that information from the TBAC meeting process is incorporated into markets in advance, which may explain why primary dealer forecasts frequently anticipate Treasury's announced issuance sizes with near-perfect accuracy.

Figure 5: U.S. Treasury 10-Year Note Yield Surprises (in %) During Quarterly Refunding Announcement high-frequency Windows Over Time



Note: The figure shows intraday changes in the 10-year Treasury yield around each QRA. Most events produce negligible yield movements, reflecting predictable issuance policy, while large positive surprises correspond to unexpected increases in duration supply. Source: Bloomberg.

Figure 6: U.S. Treasury 3-Month Bill Yield Surprises (in %) During Quarterly Refunding Announcement high-frequency Windows Over Time



Note: This figure reports intraday changes in the 3-month Treasury bill yield around each QRA. Bill yields exhibit minimal response to issuance shocks, consistent with their money-like role. Source: Bloomberg.

3.3 Treasury Quarterly Refunding Issuance Expectations

3.3.1 Primary Dealer Treasury Issuance Forecasts

For decades, certain primary dealers have published forecasts of Treasury QRA issuance across maturities in their analyst reports. For example, since the 1990s, in the days leading up to each QRA, J.P. Morgan releases a research note containing a table of projected issuance for each maturity for the horizon of at least one month following the QRA. Forecast horizons have varied across dealers and over time: in the early 2010s, some dealers published only one-month issuance projections at select maturities, whereas J.P. Morgan now provides maturity-level issuance forecasts roughly a year ahead. Other dealers do not publish forecasts in regular research reports.

Since 2020, Bloomberg has conducted surveys of primary dealers in the days preceding each QRA, publishing aggregated issuance forecasts for the upcoming three months at each maturity. These forecasts typically appear in Bloomberg news articles or on the Bloomberg Terminal (Figure A6 illustrates the Bloomberg compiled primary dealer Treasury issuance forecast table released ahead of the Q4 2023 QRA on November 1, 2023).[8]

We obtained issuance forecast data from Bloomberg back to 2020 and use the median primary dealer forecast at each maturity. In addition, we collected J.P. Morgan issuance forecasts dating back to the mid-1990s, though the format of these reports has varied over time. Prior to the mid-2010s, J.P. Morgan published only one-month-ahead issuance projections; beginning in the mid-2010s, its forecasts extended up to one year or longer.

To construct the unexpected component of issuance (the issuance surprise), we compute the difference between the duration of next-quarter issuance announced in the QRA statement by the Assistant Secretary for Financial Markets ($D_{T,T+1}$) and the duration implied by the median issuance forecast from J.P. Morgan's analyst report ($E[D_{T,T+1}]$):

$$D_{T,T+1} - E_T[D_{T,T+1}] \tag{2}$$

Assuming the forecast is an unbiased conditional expectation, we obtain an unexpected weighted-average maturity (quantity) measure to regress against high-frequency yield (price) surprises in order to estimate elasticities.

To compute unexpected changes in duration, we multiply the unexpected issuance (in dollars) at each maturity by its duration relative to the 10-year Treasury note, yielding unexpected duration in ten-year equivalents. We then scale this measure by GDP to account for growth in the economy and the overall stock of debt.

To compute unexpected changes in WAM, we multiply the unexpected issuance at each maturity by its maturity in years and divide by nominal GDP, producing a measure of the unexpected shift in WAM over the next quarter.

As an example, consider the fourth-quarter 2023 Treasury Quarterly Refunding on November 1, 2023. Primary dealer forecasts for 2-year, 3-year, 5-year, and 7-year notes ($171 billion, $150 billion, $174 billion, and $120 billion, respectively, over the next quarter) were perfectly aligned with the issuance announcement in Figure A5 and Figure A6.

However, dealers expected $117 billion, $45 billion, and $69 billion of 10-year, 20-year, and 30-year bonds, while Treasury announced only $114 billion, $42 billion, and $66 billion at these maturities. Thus, issuance was $3 billion lower than expected for each of the three long-term maturities.

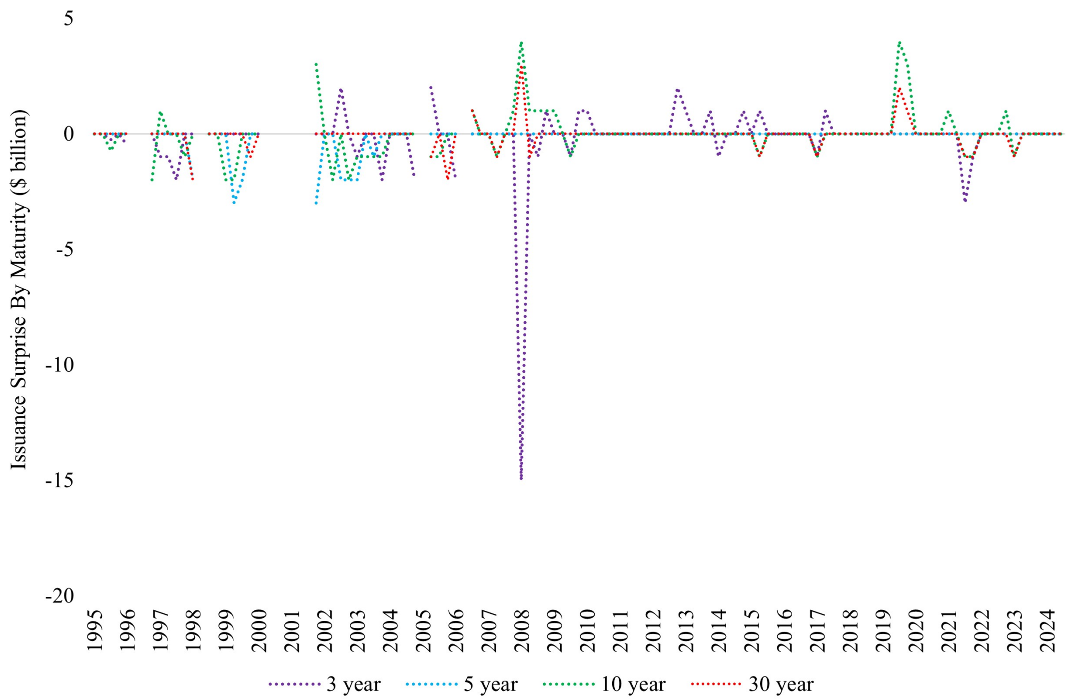
To calculate the unexpected impact on **duration** in ten-year equivalents, we multiply each unexpected issuance by its maturity in years, scale by the 10-year note's duration, and divide by nominal GDP. This is the calculation for example above:

$$\Delta D = \frac{-3 \times \frac{8.7}{8.7} \, + \, -3 \times \frac{13.35}{8.7} \, + \, -3 \times \frac{18}{8.7}}{28{,}424} = \frac{-13.81}{28{,}424} \approx -0.00049,$$

where amounts are in billions of dollars and years, scaled by GDP. Hence, the Q4 2023 refunding implied an unexpected supply of −0.049% ten-year equivalent dollars of face value as a fraction of GDP, that is, roughly half a decimal point of GDP.

[8] The authors are grateful to Elizabeth Stanton at Bloomberg for providing the quarterly Treasury issuance expectations data collected from primary dealers since 2019.

Figure 7: Issuance Shocks By Maturity Over Time (in USD billions)



Note: Issuance shocks are measured as the deviation of announced quarterly issuance duration from primary dealer expectations, in dollar billions. Large positive shocks coincide with major fiscal expansions such as the 2008 financial crisis and the 2020 pandemic response. Source: Bloomberg, J.P. Morgan, and U.S. Treasury.

To calculate the unexpected impact on **WAM**, we multiply each unexpected issuance by its maturity in years and divide by nominal GDP:

$$\Delta WAM = \frac{-3 \times 10 \ + \ -3 \times 20 \ + \ -3 \times 30}{26{,}960} = \frac{-180}{26{,}960} \approx -0.0067 \text{ years.}$$

Thus, the Q4 2023 refunding implied an unexpected WAM shift of -0.0067 years.

Figure 8 plots the time series of duration issuance shocks (measured in ten-year equivalent duration scaled by GDP). Some of the largest surprises coincide with recessions, when the Treasury adjusted maturities to finance substantial deficits—most notably during the 2008 financial crisis and the 2020 COVID-19 pandemic.

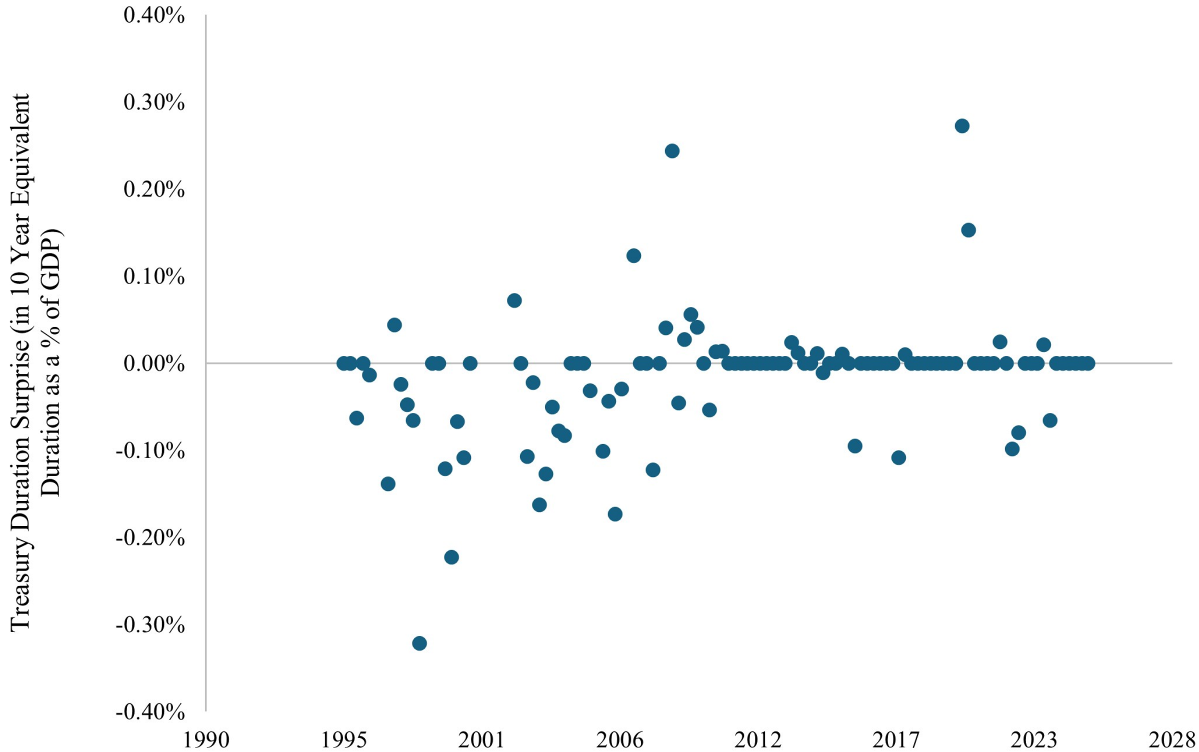
After 2011, issuance patterns became far more predictable. This reflects a shift in debt-management strategy: maturities have been adjusted more gradually, with the Treasury focusing on 3-year, 10-year, and 30-year securities and altering auction sizes only in small, incremental steps. A few exceptions stand out, such as the reintroduction of the 20-year bond in the third quarter of 2020.

By contrast, the pre-2011 era was characterized by abrupt and often unpredictable changes. In October 1998, Treasury eliminated the 3-year note and replaced it with a 5-year note. The 30-year bond, suspended in 2001, was reintroduced in February 2006, initially on a semiannual basis before later moving to quarterly issuance. The 3-year note was discontinued in November 2007, only to be revived a year later in November 2008. These sudden shifts generally subsided after the global financial crisis, as Treasury began a more stable and transparent issuance regime that has persisted since.

3.3.2 U.S. Treasury Primary Dealer Auction Size Surveys

Each quarter, prior to the QRA, the U.S. Treasury surveys primary dealers on their expectations for issuance sizes at each maturity. This survey, known as the Primary Dealer Auction Size Survey, has been conducted regularly since

Figure 8: Duration Issuance Shocks Over Time (in 10-year Equivalent Duration As A Fraction of GDP)



Note: Duration shocks are measured as the deviation of announced quarterly issuance duration (in 10-year equivalents) from primary dealer expectations, scaled by GDP. Large positive shocks coincide with major fiscal expansions such as the 2008 financial crisis and the 2020 pandemic response. Source: Bloomberg, J.P. Morgan, and U.S. Treasury.

2015Q2.

Treasury publishes only summary statistics from the survey, and only for the second and fourth quarters of each year. Since 2018Q2, the published format has included some additional detail. However, a key limitation is that the data are aggregated into means and variances of expected issuance over relatively long horizons, whereas Treasury's actual announcements concern issuance decisions for the upcoming quarter. This mismatch makes the survey difficult to align with short-term policy decisions, and therefore we do not use it in our analysis.

3.4 Empirical Approach

With both Treasury duration shocks and yield surprises in hand, we regress the high-frequency market responses to QRAs on the measured surprises in duration risk from the QRA during high-frequency windows:

$$\Delta r_T = \alpha + \beta * (D_{T,T+1} - \mathbb{E}_{T-1}[D_{T,T+1}]) + \epsilon_T \tag{3}$$

where Δr_T denotes the yield surprise, $(D_{T,T+1} - \mathbb{E}_{T-1}[D_{T,T+1}])$ is the surprise in the duration of newly announced bond supply for the next quarter, and ϵ_T is the error term. Because the J.P. Morgan duration-surprise series provides a longer history and is highly correlated with the Bloomberg survey-based series (Figure A7), we use the J.P. Morgan series in our main analysis. Supplementary results using the Bloomberg survey data are reported in Appendix Table 1.

3.5 Intraday Event–Study Regression Specification

To quantify the dynamic intraday response of Treasury yields to unexpected issuance duration, we estimate a sequence of minute-by-minute event-study regressions. Let t index U.S. Treasury Quarterly Refunding Announcement (QRA) dates (events), and let k denote the minute offset from the 08:30 ET release ($k \in [-60, 960]$ in our baseline window). Define $\Delta y_t^{10}(k)$, $\Delta y_t^{3m}(k)$, and $\Delta s_t(k)$ as the changes in the 10-year yield, 3-month yield, and their spread, respectively, relative to the last print strictly before 08:30am ET. Our measure of the issuance surprise, $(D_{T,T+1} - \mathbb{E}_{T-1}[D_{T,T+1}])$, is the deviation of announced ten-year-equivalent issuance for the coming quarter from analyst expectations, expressed in as a percent of GDP for the coming quarter.

For each minute k, we estimate the following regressions:

$$\Delta r_t^{10}(k) = \alpha_k^{10} + \beta_k^{10}\,(D_{T,T+N} - \mathbb{E}_{T-1}[D_{T,T+1}]) + \varepsilon_{t,k}^{10}, \tag{4}$$

$$\Delta r_t^{3m}(k) = \alpha_k^{3m} + \beta_k^{3m}\,(D_{T,T+1} - \mathbb{E}_{T-1}[D_{T,T+1}]) + \varepsilon_{t,k}^{3m}, \tag{5}$$

$$\Delta(r_t^{10y} - r_t^{3m})(k) = \alpha_k^{s} + \beta_k^{s}\,(D_{T,T+1} - \mathbb{E}_{T-1}[D_{T,T+1}]) + \varepsilon_{t,k}^{s}. \tag{6}$$

Standard errors are clustered by event date. The sequence of coefficients $\{\beta_k^{\cdot}\}$ across k traces the cumulative response (in basis points) to a one–percentage-point-of-GDP duration surprise.

Let r_0^{10y} and r_0^{3m} denote the last transaction strictly before the announcement, at 08:29 am ET on the event day for the U.S. Treasury 10-year bond yield and 3-month bill yield respectively. We define

$$\Delta r_t^{10y}(k) \equiv r_t^{10}(k) - r_0^{10}, \tag{7}$$

$$\Delta r_t^{3m}(k) \equiv r_t^{3m}(k) - r_0^{3m}, \tag{8}$$

$$\Delta(r_t^{10y} - r_t^{3m})(k) \equiv \Delta r_t^{10}(k) - \Delta r_t^{3m}(k). \tag{9}$$

An equivalent way to write (4)–(6) in one regression is

$$\Delta r_t(k) = \sum_{m \in \mathcal{K}} \mathbf{1}\{k = m\}\big(\alpha_m + \beta_m(D_{T,T+1} - \mathbb{E}_t[D_{T,T+1}])\big) + \varepsilon_{t,k}, \qquad \mathcal{K} = \{-60, \ldots, 960\}, \tag{10}$$

which delivers the same β_m coefficients while estimating all minutes jointly.

3.6 Local Supply Effects by Maturity

Our baseline specification summarizes Treasury issuance news using a single scalar duration shock. To allow for maturity-specific supply effects and cross-maturity transmission along the yield curve, we also estimate a more flexible specification that treats issuance surprises at different maturities as distinct shocks.

Let t index Quarterly Refunding Announcement (QRA) events, and let $\tau \in \{3, 10, 30\}$ denote the maturity of the Treasury yield. For each event, we measure high-frequency yield changes $\Delta y_t(\tau)$ in a narrow window around the QRA. We construct issuance surprises at the 3-year, 10-year, and 30-year maturities, denoted by $s_t^{(3)}$, $s_t^{(10)}$, and $s_t^{(30)}$, defined as the difference between announced issuance and market expectations immediately prior to the announcement. All issuance surprises are scaled into comparable risk units (in ten year equivalents) so that coefficients are interpretable across maturities.

For each yield maturity τ, we estimate the following event-study regression:

$$\Delta y_t(\tau) = \alpha(\tau) + b_3(\tau)\, s_t^{(3)} + b_{10}(\tau)\, s_t^{(10)} + b_{30}(\tau)\, s_t^{(30)} + u_t(\tau), \tag{11}$$

where $u_t(\tau)$ is an error term. Equation (11) is estimated separately for $\tau = 3, 10$, and 30 years.

The coefficients $b_j(\tau)$ capture the partial effect of an issuance surprise at maturity j on the yield at maturity τ, holding fixed issuance surprises at other maturities. This specification therefore allows for both maturity-specific ("local") supply effects, reflected in the diagonal elements $b_j(j)$, and cross-maturity spillovers, reflected in the off-diagonal elements $b_j(\tau)$ for $j \neq \tau$. Spillovers may arise through arbitrage, hedging, and portfolio rebalancing along the yield curve, even if the initial supply shock is concentrated at a particular maturity.

Inference is conducted using heteroskedasticity-robust standard errors at the event level. Throughout, we interpret the estimated coefficient matrix $\{b_j(\tau)\}$ as describing the deformation of the yield curve induced by maturity-specific supply news, without imposing additional restrictions on the transmission of shocks across maturities.

4 Reduced-Form high-frequency Identification Results

4.1 U.S. Treasury Yields and Term Spreads

Relatively few QRAs elicit market reactions greater than one basis point, suggesting that market participants' expectations are typically well aligned with Treasury issuance plans by the time of the announcement. This may reflect Treasury's policy of 'regular and predictable' issuance as well as potential information leakage during the TBAC consultation process, consistent with the 'pre-announcement effect' documented by Wang and Zhao (2024).

Two exceptions stand out. On May 6, 2020—the first refunding following the onset of the COVID-19 pandemic and passage of the CARES Act—Treasury announced heavier reliance on long-term bonds relative to short-term bills by announcing the issuance of more 20-year bonds than was anticipated. This announcement raised long-term yields by roughly 3 basis points, while short-term bill yields remained unchanged, as shown in Figure 9. On November 1, 2023, Treasury announced increased issuance of short-term bills and reduced issuance of longer-term notes and bonds. This shift lowered long-term yields by about 5 basis points, again leaving short-term bill yields largely unaffected, as shown in Figure 10, in line with the preferred-habitat hypothesis of Vayanos and Vila (2021).

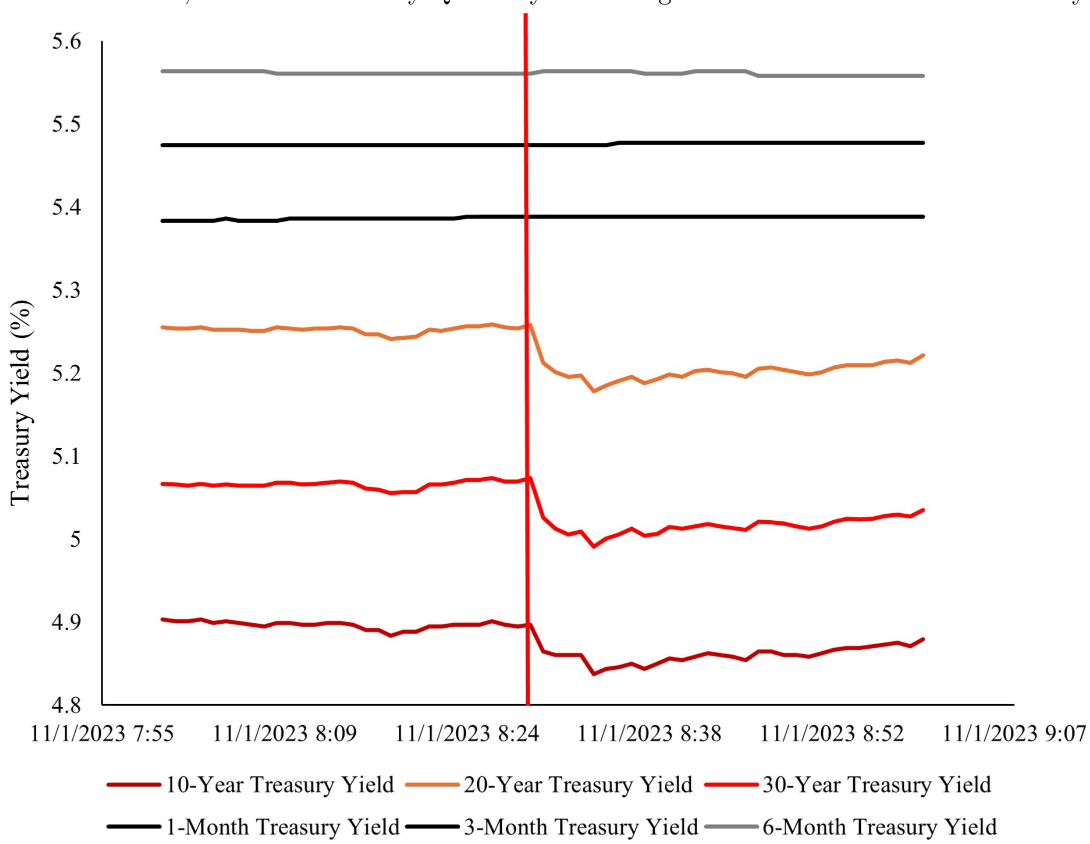
We quantify these dynamics by regressing high-frequency yield changes on the unexpected shift in Treasury duration risk implied by issuance over the next quarter. Table 1 reports the results. The 3-month to 10-year term spread term spread increases by 0.072% for each 1% of GDP in unexpected ten-year-equivalent duration, significant at the 5% level (Figure 11). The 10-year yield rises by 0.081% per 1% of GDP in unexpected duration, significant at the 1% level (Figure 12). By contrast, the 3-month yield responds by only 0.008% (Figure 13), which is not statistically significant, indicating that short-term rates are largely insensitive to additional duration supply.

Several caveats are in order. QRAs pertain to issuance over the next quarter but in practice signal Treasury's issuance policy over a much longer horizon, since issuance paths change infrequently. Thus, our measured three-month surprises likely understate the effective duration shock, biasing upward the estimated yield elasticities. Moreover, our regressions identify local, linear responses; if Treasury demand is nonlinear, the marginal effect of duration shocks could differ at other levels of debt composition.

Even so, our elasticities are larger than those reported in prior work. Wright (2022), for example, regresses the ten-year term premium of Adrian et al. (2013) on the maturity-weighted debt-to-GDP ratio of Greenwood and Vayanos (2014) using monthly data from 1961–2007 and finds an effect of 0.32–0.34% per year of weighted average maturity which is approximately 0.06% per 1% of ten year equivalent (TYE) duration. Belton et al. (2018) report similar magnitudes.

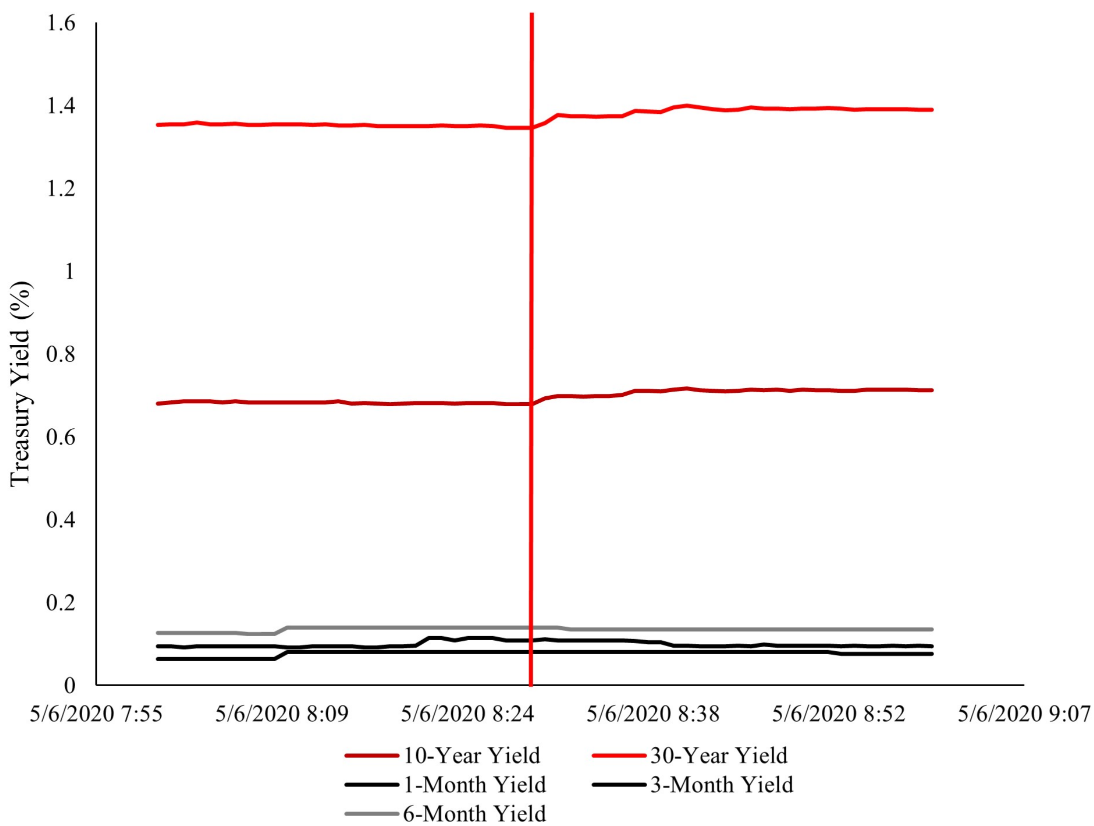
Nonetheless, the relative magnitudes we estimate—large effects on long-term yields and negligible effects on short-term bills—are directionally consistent with models in which term premia respond to duration supply shocks while short-term debt is absorbed as a money-like asset, such as Greenwood, Hanson, and Stein (2015). These results imply that shifting issuance toward short-term bills and away from long-term bonds can reduce borrowing costs, with direct implications for optimal debt management.

Figure 9: November 1, 2023 U.S. Treasury Quarterly Refunding Announcement Effect On Treasury Yields



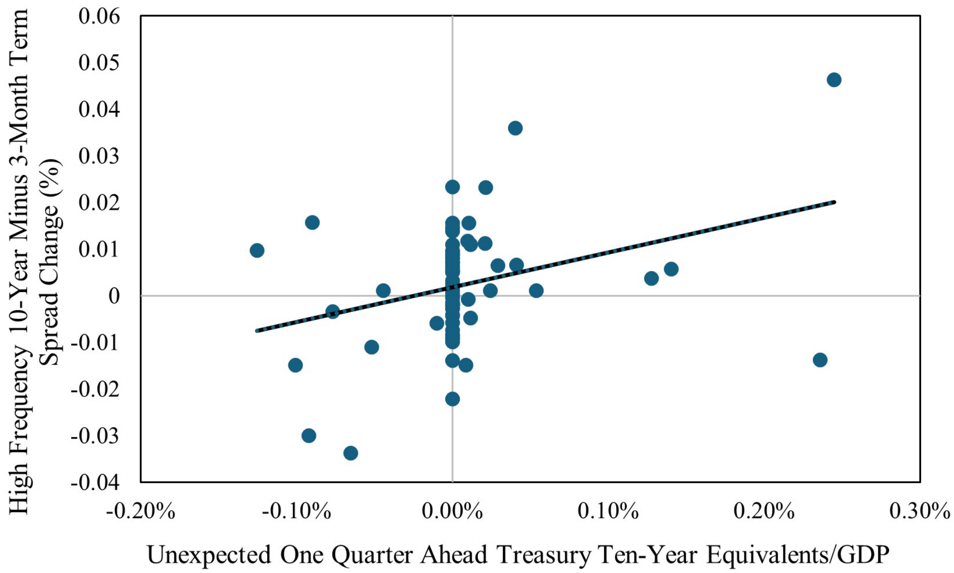
Note: High-frequency intraday reaction of Treasury yields around the November 2023 U.S. Treasury Quarterly Refunding Announcement (QRA), when Treasury announced increased bill issuance and reduced long-term issuance. Long-term yields fell by roughly 5 basis points, consistent with reduced duration supply.

Figure 10: May 6, 2020 U.S. Treasury Quarterly Refunding Announcement Effect On Treasury Yields



Note: Intraday yield response to the May 2020 QRA, the first after the onset of COVID-19. Treasury announced heavier reliance on long-term bonds, raising 10-year yields by roughly 3 basis points. Short-term bill yields were unchanged.

Figure 11: 10-Year Treasury Yield Minus 3-Month Treasury Yield Market Reactions To Quarterly U.S. Treasury Refunding Announcement Versus Deviation From Treasury Issuance Expectations in Ten-year Equivalent Duration



Note: Scatter plot of the 3-month to 10-year term spread change in high-frequency windows around U.S. Treasury Quarterly Refunding Announcements (QRAs) against unexpected duration issuance (in ten-year equivalents as a percent of GDP). A one-percentage-point increase in duration supply raises the term spread by about 7 basis points.

Table 1: U.S. Treasury Quarterly Refunding high-frequency Yield Impacts Versus Unexpected One Quarter Ahead Shift in Treasury Duration

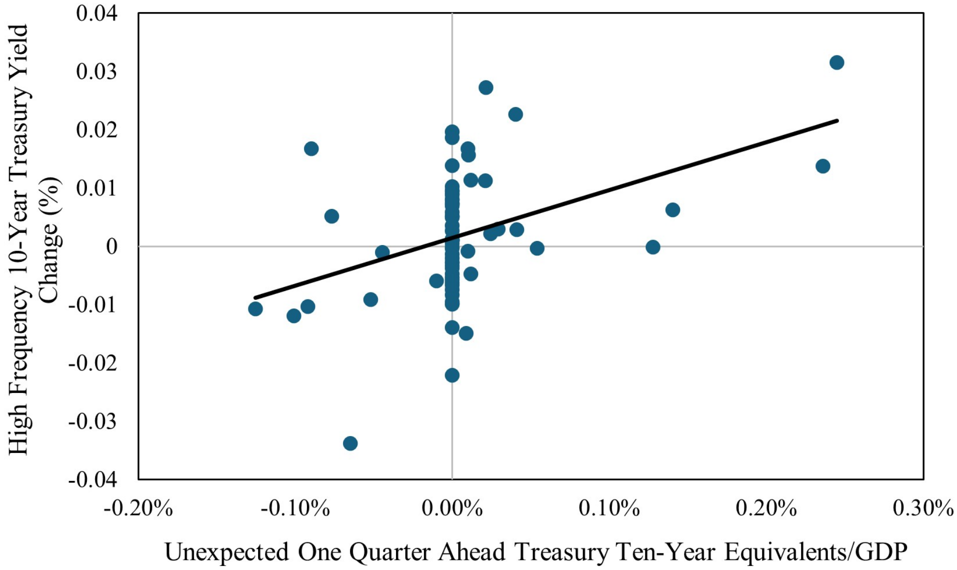
	U.S. Treasury Quarterly Refunding		
	high-frequency (30-minute Window) Change In Yield (%)		
	10-Y minus 3-M yield (bps)	10-Y yield (bps)	3-M yield (bps)
Duration Shock (1 pp of GDP)	7.463**	8.207***	0.879
	(2.288)	(1.698)	(3.809)
Constant	0.000	0.000	0.000
	(0.002)	(0.002)	(0.001)
R-Square	0.174	0.109	0.004
N	72	72	72

Notes: * $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$

Notes: Regressions of high-frequency changes in 3-month to 10-year term spread, 10-year yields, and 3-month bond yields around U.S. Treasury Quarterly Refunding Announcements (QRAs) against unexpected duration issuance (in ten-year equivalents as a percent of GDP). Duration is measured in one quarter ahead Treasury Ten-Year Equivalents as a Fraction of GDP. Standard errors are Newey–West heteroskedasticity robust standard errors.

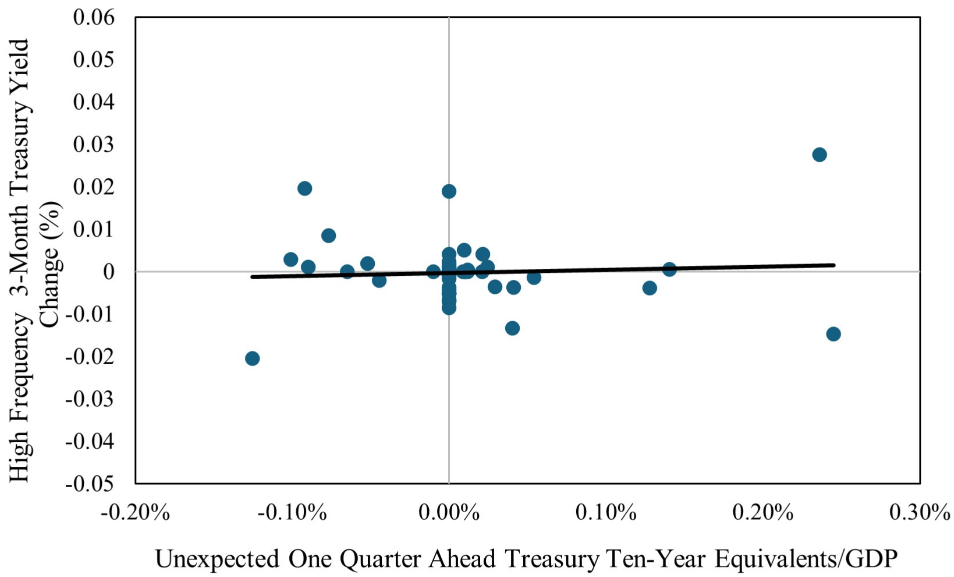
Our finding that short-term bills appear relatively insensitive to supply shocks suggests that their moneyness may have an important role in the pricing of the yield curve. This interpretation is consistent with models such as Greenwood et al. (2015), in which money enters the utility function and short-term bills are treated as money-like assets. A possible implication for debt management is that shifting issuance toward short-term bills and away from

Figure 12: 10-Year Treasury Yield Market Reactions To Quarterly U.S. Treasury Refunding Announcement Versus Deviation From Treasury Issuance Expectations in Ten-year Equivalent Duration



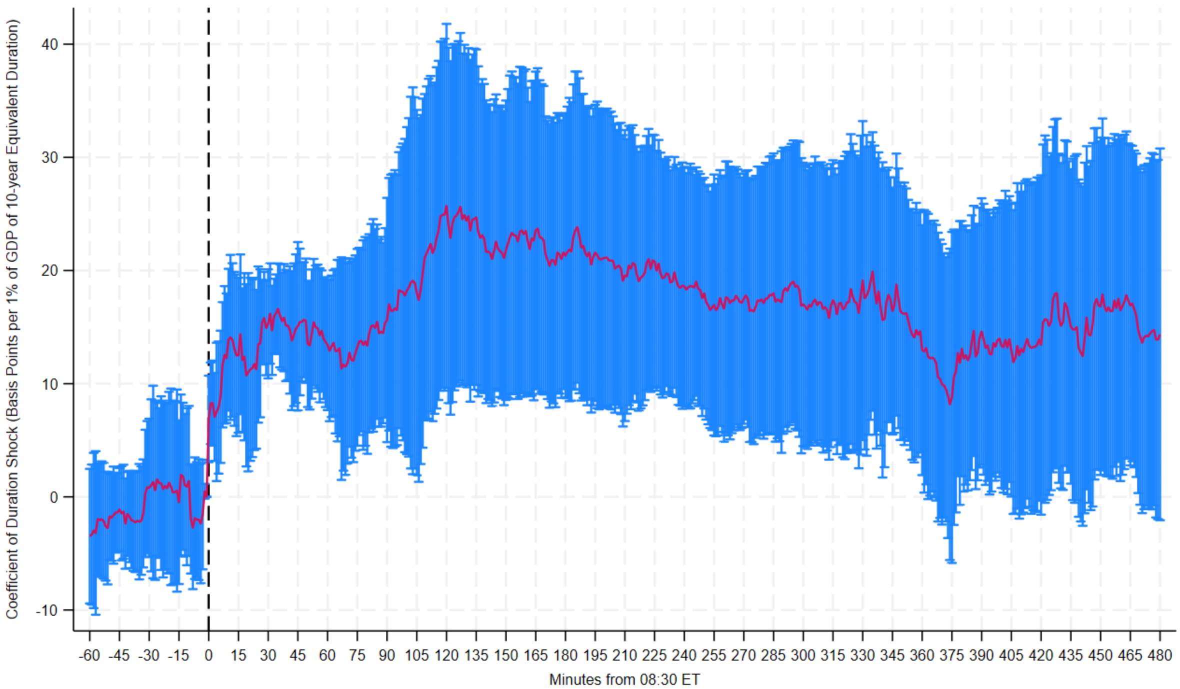
Note: Scatter plot of 10-year yield changes versus unexpected issuance duration. Long-term yields rise roughly 8 basis points per 1% of GDP surprise in duration supply.

Figure 13: 3-Month Treasury Yield Market Reactions To Quarterly U.S. Treasury Refunding Announcement Versus Deviation Treasury Issuance Expectations in Ten-year Equivalent Duration



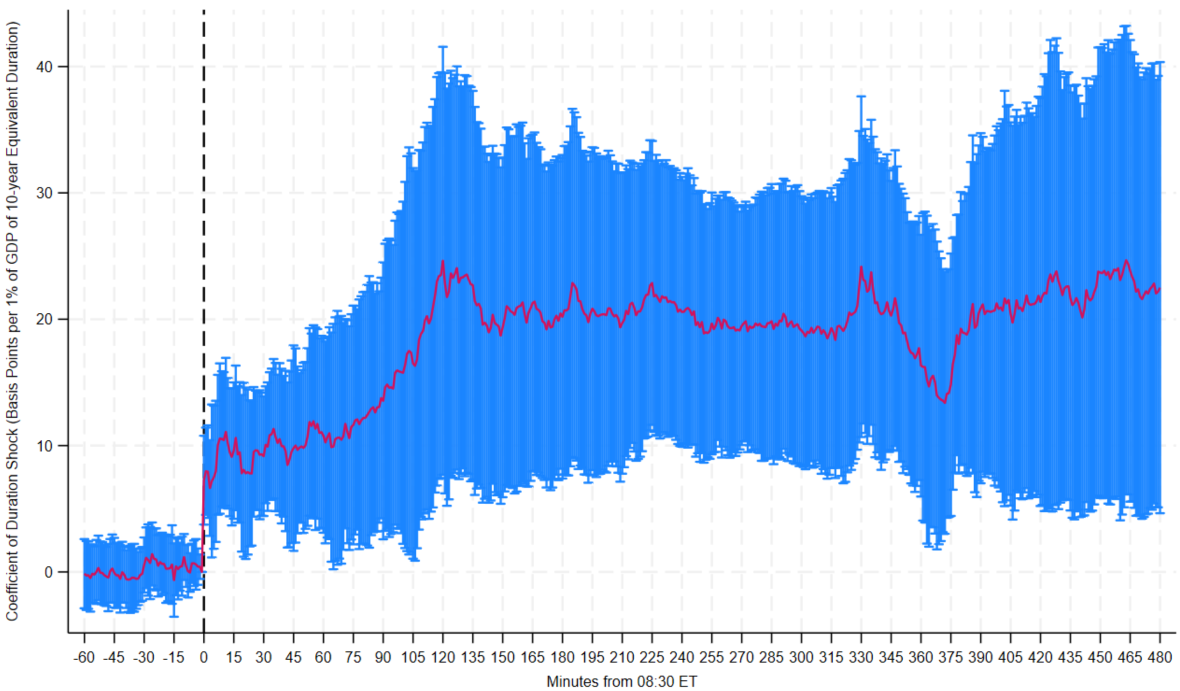
Note: The 3-month Treasury yield shows negligible response to duration shocks, suggesting that short-term debt behaves as a money-like asset largely insulated from maturity-mix surprises.

Figure 14: Intraday Event Study Plot of 10-Year Treasury Yield Minus 3-Month Treasury Yield Cumulative Market Reaction To 1% of GDP Surprise in 10-Year Equivalent Duration At U.S. Treasury Quarterly Refunding Announcement



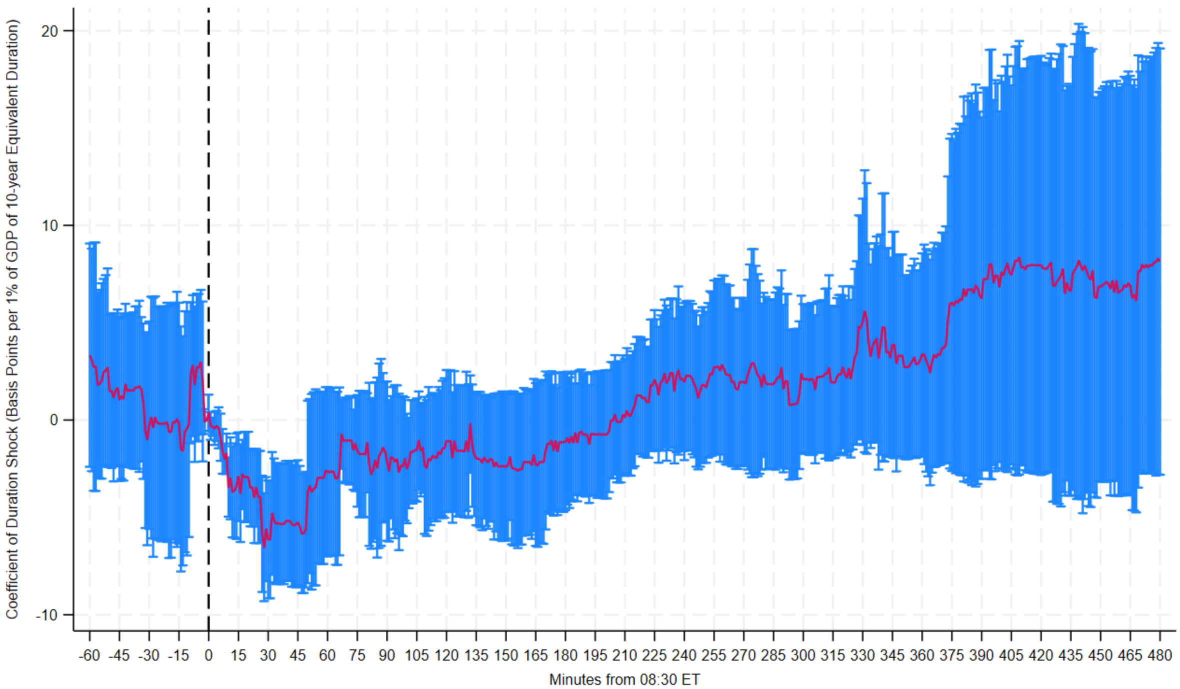
Note: Notes: Daily aggregated event study plot showing the cumulative market reaction of the 10-year minus 3-month Treasury yield term spread to a 1% of GDP surprise in 10-year equivalent duration at U.S. Treasury Quarterly Refunding Announcements over a 1-day horizon. The red line represents coefficient estimates from regressions of yield changes (relative to pre-announcement levels at 8:29 AM ET) on duration shocks, aggregated to daily frequency. The blue shaded area shows 95% confidence intervals based on cluster-robust standard errors (clustered by event). The vertical dashed line at zero marks the announcement time.

Figure 15: Intraday Event Study Plot of 10-Year Treasury Yield Cumulative Market Reaction To 1% of GDP Surprise in 10-Year Equivalent Duration At U.S. Treasury Quarterly Refunding Announcement



Notes: Daily aggregated event study plot showing the cumulative market reaction of the 10-year Treasury yield term spread to a 1% of GDP surprise in 10-year equivalent duration at U.S. Treasury Quarterly Refunding Announcements over a 1-day horizon. The red line represents coefficient estimates from regressions of yield changes (relative to pre-announcement levels at 8:29 AM ET) on duration shocks, aggregated to daily frequency. The blue shaded area shows 95% confidence intervals based on cluster-robust standard errors (clustered by event). The vertical dashed line at zero marks the announcement time.

Figure 16: Intraday Event Study Plot of 3-Month Treasury Yield Cumulative Market Reaction To 1% of GDP Surprise in 10-Year Equivalent Duration At U.S. Treasury Quarterly Refunding Announcement



Notes: Daily aggregated event study plot showing the cumulative market reaction of the 3-month Treasury yield term spread to a 1% of GDP surprise in 10-year equivalent duration at U.S. Treasury Quarterly Refunding Announcements over a 1-day horizon. The red line represents coefficient estimates from regressions of yield changes (relative to pre-announcement levels at 8:29 AM ET) on duration shocks, aggregated to daily frequency. The blue shaded area shows 95% confidence intervals based on cluster-robust standard errors (clustered by event). The vertical dashed line at zero marks the announcement time.

longer maturities may generate fiscal savings, as term premia adjust to surprises in the supply of duration risk and the demand for money-like instruments.

These results may also be consistent with Campbell (1995), who observed that when long rates are high relative to short rates (e.g., when the yield curve is not inverted), short rates tend to rise as implied by the expectations hypothesis, but long rates tend to fall—contrary to the expectations hypothesis. This pattern suggests that debt managers may have an incentive to shorten maturities when the yield curve is steep in order to minimize borrowing costs.

4.2 Local Treasury Supply Effects

Table 2: Maturity-Specific Issuance Shocks and Local Yield Responses

	1m	3m	6m	12m	2y	3y	5y	7y	10y	20y	30y
3-Year Issuance Shock	-0.00512^{***}	-0.00723^{***}	-0.00426^{***}	-0.00443^{***}	-0.00609^{***}	-0.00542^{***}	−0.00157	0.00205	0.00407^{***}	0.00168	0.00487^{**}
	(0.00083)	(0.00126)	(0.00113)	(0.00140)	(0.00144)	(0.00172)	(0.00170)	(0.00144)	(0.00152)	(0.00370)	(0.00202)
10-Year Issuance Shock	−0.00142	−0.00168	-0.00180^{**}	0.00095	0.00165	0.00272	0.00277	0.00312	0.00330	0.00277	0.00329
	(0.00121)	(0.00120)	(0.00073)	(0.00207)	(0.00302)	(0.00285)	(0.00260)	(0.00225)	(0.00218)	(0.00304)	(0.00294)
30-Year Issuance Shock	0.00109	−0.00029	0.00128	−0.00047	−0.00074	−0.00076	0.00003	0.00127	0.00232	0.00204	0.00421^{*}
	(0.00103)	(0.00103)	(0.00087)	(0.00170)	(0.00207)	(0.00193)	(0.00175)	(0.00165)	(0.00177)	(0.00325)	(0.00250)
Observations	75	75	75	75	75	75	75	75	75	45	75
R^2	0.338	0.367	0.255	0.156	0.186	0.163	0.072	0.086	0.124	0.054	0.169

Notes: The table reports event-level regressions of high-frequency changes in Treasury yields around U.S. Treasury Quarterly Refunding Announcements on issuance surprises at the 3-year, 10-year, and 30-year maturities. Columns correspond to Treasury-bill–relative yield measures at the indicated maturities. Issuance surprises are scaled into comparable risk units (ten-year-equivalent duration). Heteroskedasticity-robust standard errors are reported in parentheses. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.10$.

Table 2 reports estimates of equation (11), which allows issuance surprises at different maturities to affect Treasury yields both directly and through cross-maturity transmission. Two clear patterns emerge. First, issuance surprises at the 3-year maturity have a large and statistically significant effect on the 10-year yield, comparable in magnitude to the effect of issuance surprises at the 10-year maturity itself. Second, issuance surprises at the 30-year maturity have no statistically detectable effect on the 10-year yield once issuance at shorter maturities is taken into account.

This pattern implies economically meaningful cross-maturity transmission that is local rather than proportional to aggregate duration. Supply shocks concentrated at short and intermediate maturities spill over strongly to the 10-year yield, consistent with arbitrage, hedging, and portfolio rebalancing across nearby maturities. By contrast, long-end issuance does not independently price the 10-year yield.

Under frictionless aggregate-duration models, such as Greenwood and Vayanos (2014), issuance shocks at all maturities would load proportionally on intermediate yields once scaled by duration. The absence of such loading for 30-year issuance instead points to segmented demand and imperfect arbitrage across maturities, consistent with preferred-habitat frameworks such as Vayanos and Vila (2021).

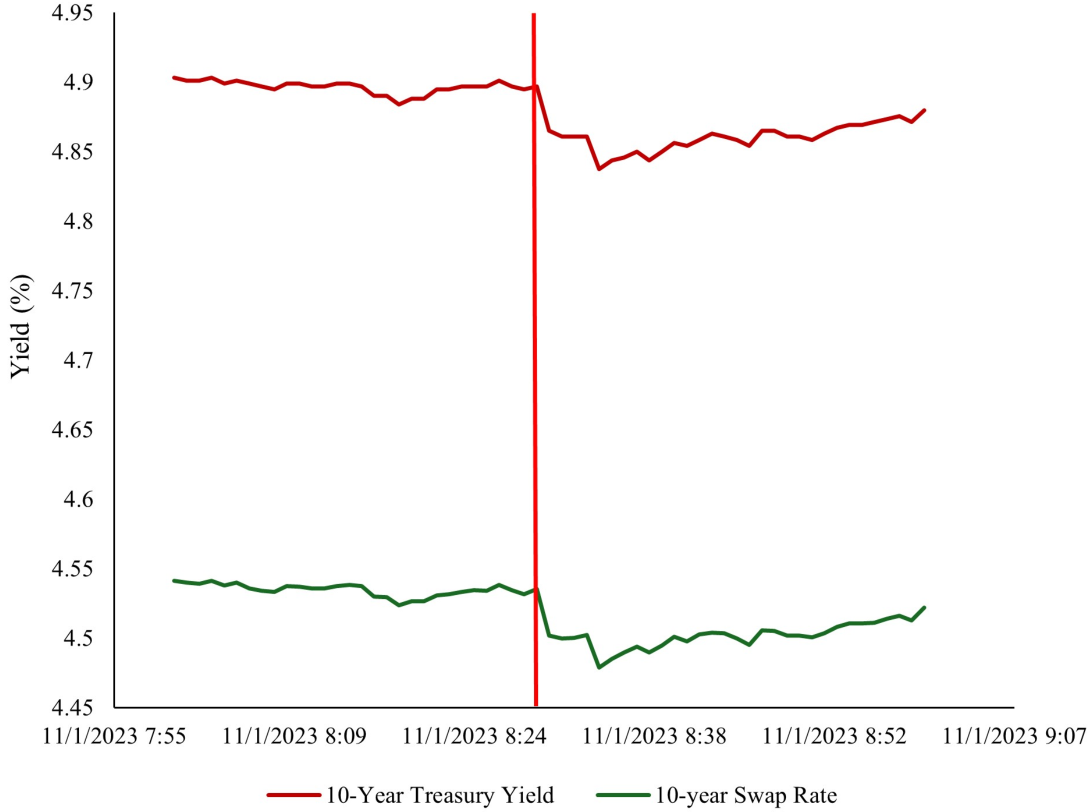
Together with the finding that short-maturity issuance significantly affects short-term yields, these results suggest that Treasury bills and near-bills behave as money-like assets with highly elastic demand, while longer-maturity yields respond primarily through portfolio rebalancing rather than direct price pressure. Overall, maturity-specific supply shocks generate both local and spillover effects along the yield curve, in line with preferred-habitat models of the Treasury market.

4.3 Swap Spreads

We also examine how swap rates and swap spreads respond in high-frequency windows around Treasury refunding announcements and how these moves correlate with issuance-duration surprises.

Like 10-year Treasury yields, 10-year swap rates rise with the size of the duration shock and statistically significantly at the 5% level (Figure 20). The estimated coefficient is 5.36 basis points for swaps for every 1% of GDP of ten

Figure 17: November 1, 2023 U.S. Treasury Quarterly Refunding Announcement Effect On Swap Rate and Treasury Yields

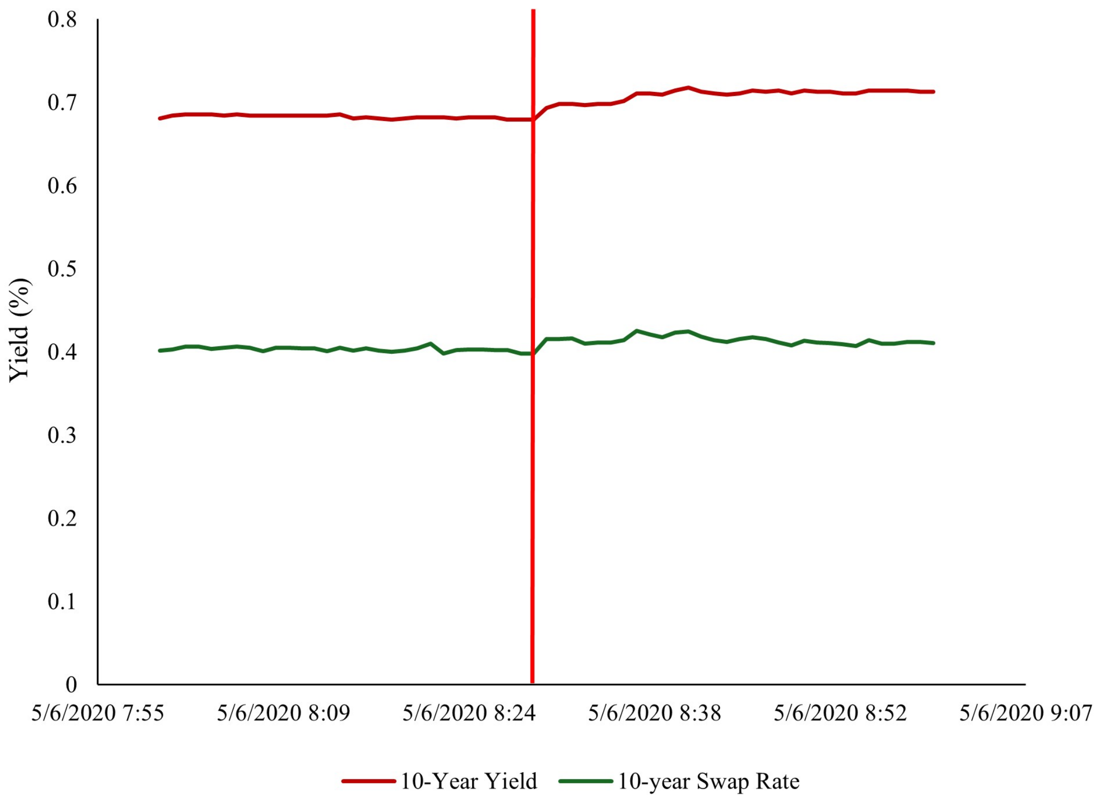


Note: Comparison of intraday changes in 10-year Treasury yields and 10-year swap rates around the November 2023 U.S. Treasury Quarterly Refunding Announcement (QRA). Both rates decline with reduced duration supply, though Treasuries react more strongly, widening swap spreads slightly.

year-equivalent duration surprise, compared with 8.21 basis points for Treasuries, suggesting that cash Treasury bonds are more sensitive to duration shocks than interest rate swaps. This difference points to limits to arbitrage and the role of intermediaries in transmitting debt management effects as well as the potential presence of clientele effects.

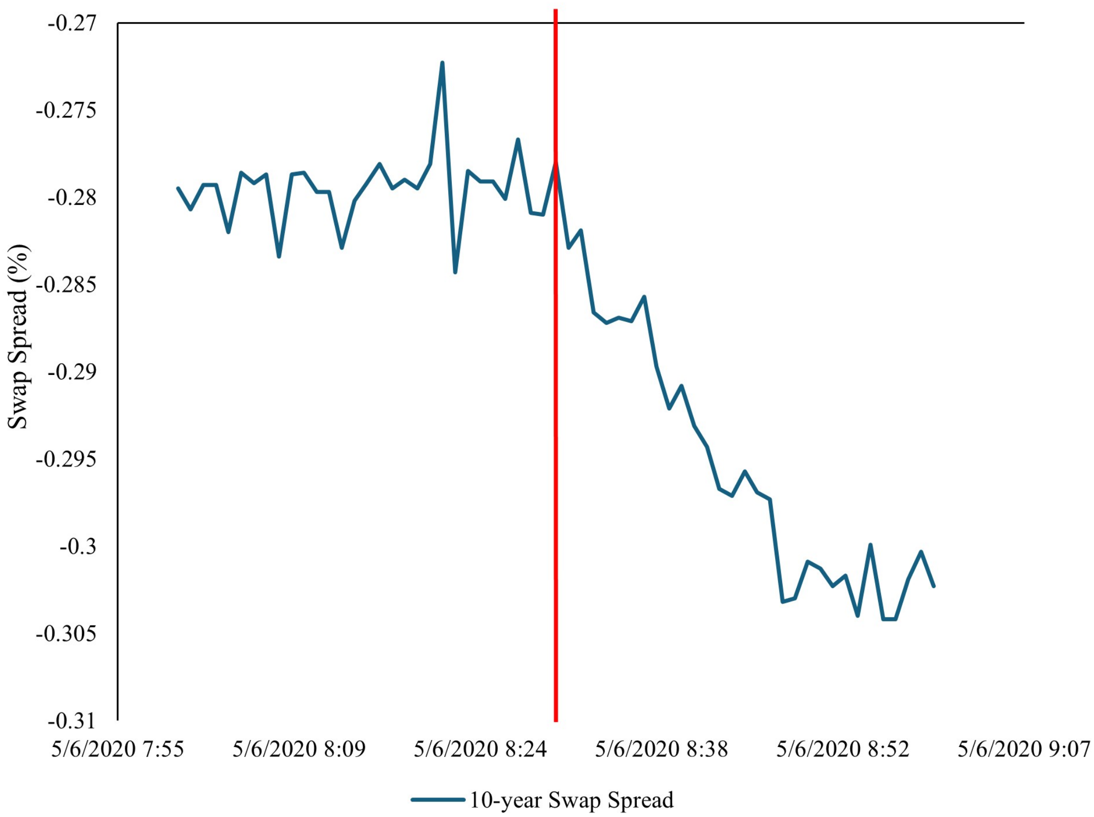
Consistent with this, 10-year swap spreads decline by -4.06 basis points for every 1% of GDP of ten year-equivalent duration surprise, though the effect is only statistically significant at the 10% level. One notable case is the Q2 2020 refunding (Figure 18 and Figure 19), when a positive duration shock led the 10-year government bond yield to rise by 3.15 basis points, the swap rate by 1.28 basis points, and the swap spread to narrow by about 1.88 basis points.

Figure 18: May 6, 2020 U.S. Treasury Quarterly Refunding Announcement Effect On Swap Rate and Treasury Yields



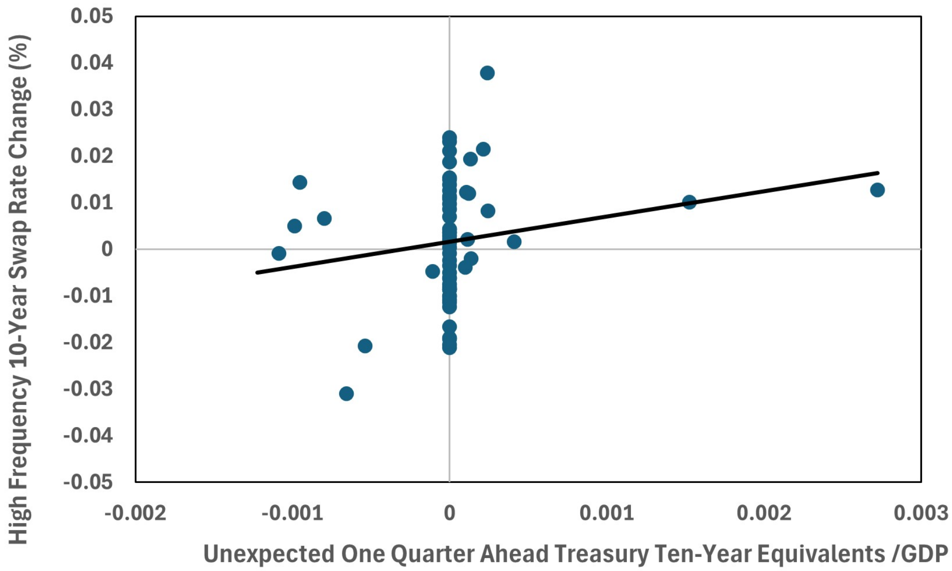
Note: Intraday changes in Treasury yields and swap rates during the May 2020 U.S. Treasury Quarterly Refunding Announcement (QRA). Long-term yields and swaps rose following unexpected long-term issuance, narrowing swap spreads.

Figure 19: May 6, 2020 U.S. Treasury Quarterly Refunding Announcement Effect On Swap Spreads



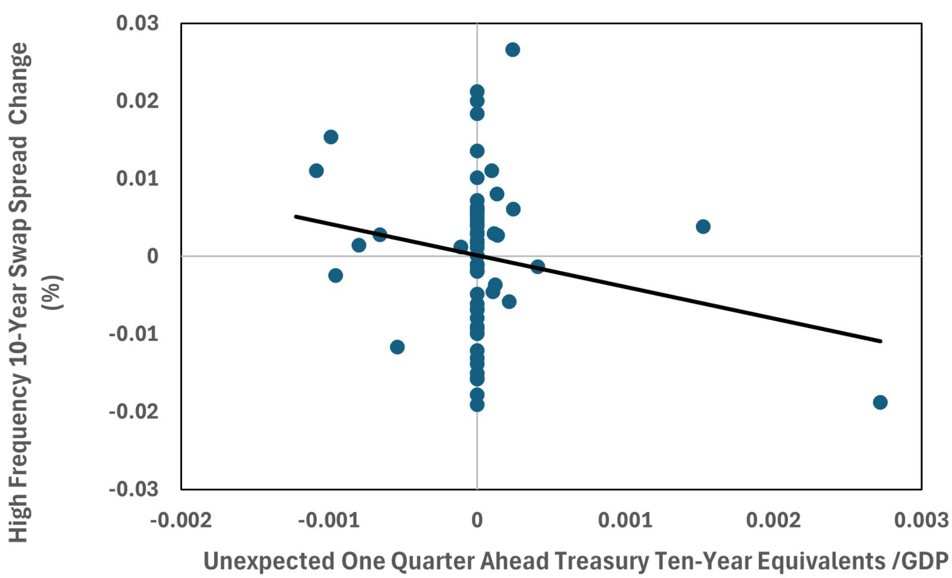
Note: The figure shows the high-frequency narrowing of 10-year swap spreads during the May 2020 U.S. Treasury Quarterly Refunding Announcement (QRA). The decline reflects increased Treasury supply and limits to arbitrage between Treasuries and swaps.

Figure 20: 10-Year Swap Rate Reactions To Quarterly U.S. Treasury Refunding Announcement Versus Deviation From 3-Month Ahead Treasury Issuance Expectations in ten-year equivalent Duration



Note: Scatter plot of high-frequency 10-year swap rate changes versus duration surprises during U.S. Treasury Quarterly Refunding Announcements (QRAs). Swap rates rise with greater duration issuance, though the response is smaller than for Treasuries, suggesting market segmentation or intermediary constraints.

Figure 21: 10-Year Swap Spread Reactions To Quarterly U.S. Treasury Refunding Announcement Versus Deviation From 3-Month Ahead Treasury Issuance Expectations in ten-year equivalent Duration



Scatter plot of high-frequency 10-year swap spread (10-year swap yield minus 10-year Treasury bond yields) changes versus duration surprises during U.S. Treasury Quarterly Refunding Announcements (QRAs). Swap spreads react negatively to duration surprises as swap rates rise with greater duration issuance more so than Treasuries, suggesting market segmentation or intermediary constraints.

Table 3: U.S. Treasury Quarterly Refunding High-Frequency Swap Spread Impacts Versus Unexpected One Quarter Ahead Shift in Treasury Duration

	U.S. Treasury Quarterly Refunding		
	high-frequency (30-minute Window) Change In Swap/Rate Yield (%)		
	10-Y Swap Spread (bps)	10-Y Swap Rate (bps)	10-Y Treasury Yield (bps)
Duration Shock (1 pp of GDP)	-4.056*	5.358**	8.207***
	(2.203)	(2.351)	(1.698)
Constant	0.000	0.002	0.000
	(0.001)	(0.002)	(0.002)
R-Square	0.038	0.032	0.109
N	62	62	72

Notes: * $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$

Notes: Duration is measured in one quarter ahead Treasury Ten-Year Equivalents as a Fraction of GDP. Standard errors are Newey–West heteroscedasticity robust standard errors.

4.4 Robustness: Overlaps with BEA GDP Releases and GDP-Surprise Controls

A small number of Quarterly Refunding Announcements (QRAs) coincide exactly in time with the Bureau of Economic Analysis (BEA) advance GDP release (both at 8:30 a.m. ET). Table 4 lists every such overlap in our sample. Because GDP is a first-order macro release, these dates could—in principle—co-move yields independently of refunding news. The events are rare and clustered at month-end Wednesdays, but to be conservative we explicitly measure and control for the GDP surprise (advance print minus the contemporaneous consensus forecast) on these days. Our results are essentially unchanged when controlling for GDP surprises during QRA announcements.

Appendix Table 6 augments our baseline high-frequency regressions with the GDP-surprise control. The coefficient on Duration Shock remains large and statistically significant for the 10-year yield (8.235 with standard error of 2.151 and p-value < 0.01) and for the 3-month to 10-year term spread (7.444 with standard error of 2.572 and p-value < 0.05), while the 3-month yield remains small and insignificant (0.790 with standard error of 1.371). The GDP-surprise coefficients are economically tiny and statistically indistinguishable from zero across all three outcomes (0.003, –0.002, and 0.005, respectively). R-squared values move only modestly relative to the baseline

The invariance of the duration-shock coefficients to GDP-surprise controls reinforces our identification: Treasury yield reactions on QRA mornings primarily reflect relative-supply information (maturity composition) and are not biased by concurrent macro-news which occurs on a limited number of days in our sample (also days where the duration shocks were largely anticipated). Consistent with the preferred-habitat view, long-maturity yields and the term spread absorb duration shocks, while bills largely are invariant. The stability of these estimates continue to support the policy implications for debt-maturity choice which come from the models.

Table 4: Overlaps of Treasury Quarterly Refunding Announcements and BEA GDP "Advance" Releases

Date (ET)	Forecast (%)	GDP Advance (%)	Surprise (pp)
2008-04-30	0.2	0.6	+0.4
2009-04-30	-4.9	-6.1	-1.2
2013-07-31	1.0	1.7	+0.7
2019-10-31	1.6	1.9	+0.3
2024-10-31	3.1	2.8	-0.3
2025-04-30	0.4	-0.3	-0.7
2025-07-31	2.3	3.0	+0.7

Notes: Dates are the same-morning QRA (8:30am ET, Wednesday) and BEA GDP "advance" prints. Surprise is computed as the actual GDP print minus the median financial analyst forecast (percentage points).

5 Mean Variance Optimal Government Debt Management

Next, we use our reduced-form estimates of the effect of additional duration on the term spread, identified with high-frequency data, to inform optimal debt management within the mean–variance framework of Belton et al. (2018), originally developed for and is still used by the Treasury Borrowing Advisory Committee (TBAC) to inform Treasury issuance policy. Because our reduced-form estimates imply a duration–term-spread elasticity of roughly 8–12 basis points per 1% of GDP, which is substantially larger than the 6 bp benchmark used in Belton et al. (2018), the efficient frontier rotates inward and increases the attractiveness of bill issuance.

The mean–variance framework is particularly well-suited to the institutional setting of U.S. debt management because it formalizes the real-world tradeoff between expected borrowing costs and refinancing risk faced by the TBAC and the Office of Debt Management. Unlike Ramsey models in which the planner internalizes future tax distortions, this framework reflects that Treasury policy is executed under political, budgetary, and market-functioning constraints rather than full intertemporal welfare maximization.

In this framework, the debt manager's objective is to minimize debt service costs (as a share of GDP) subject to a constraint on the volatility of debt service. In general, short-term government debt has higher variance in interest expenses over time compared to longer-term debt due to monetary policy and macroeconomic uncertainty but a lower average interest expense due to the risk premia associated with long-term debt.

New issuance in the model has 4 policy variables (kernels) for new issuance (Bills, Belly, Bonds, and TIPS) each of which scales all maturities within the particular policy choice block. Nominal bonds have less liquidity risk and more inflation risk compared to TIPS. Data from the Monthly Statement of the Public Debt (MSPD) is used to detail the outstanding issuance.

5.1 Debt Manager Objective Function

The government debt manager chooses the maturity composition of issuance to minimize a mean–variance loss function defined over long-run fiscal outcomes. Let p index simulated paths of the macroeconomic and interest-rate environment. The debt manager solves

$$\min_{\{\text{issuance policy}\}} \sum_p \left[\Omega_p + \lambda\, \varepsilon_p^+ + \lambda\, \varepsilon_p^-\right], \tag{12}$$

where Ω_p denotes the interest cost-to-GDP ratio in path p at a fixed long-run horizon, and $\lambda \geq 0$ captures the debt manager's degree of risk aversion.

The deviation of path-p outcomes from the cross-path mean is defined as

$$\varepsilon_p^+ - \varepsilon_p^- = \frac{(C_{pt} + D_{pt})Z_{pt} - \widehat{\Omega}}{GDP_{pt}}, \qquad t = 20 \text{ years ahead}, \tag{13}$$

where C_{pt} denotes coupon payments, D_{pt} denotes principal repayments, Z_{pt} maps debt service into effective interest costs, and GDP_{pt} is gross domestic product along path p.

The cross-path average interest cost is given by

$$\widehat{\Omega} = \frac{1}{P}\sum_{p=1}^{P}(C_{pt} + D_{pt})Z_{pt}, \qquad t = 20 \text{ years ahead}. \tag{14}$$

Positive and negative deviations from the mean are penalized symmetrically through ε_p^+ and ε_p^-.

5.2 Debt Management Constraints

The constraints in the convex optimization problem include a number of macroeconomic rules of thumb, which help produce simulations of the macroeconomy over 20 years. The debt manager optimizes in a one-shot fixed optimization problem given these simulations.

These constraints include several standard components relevant to debt management, including a macroeconomic block, a rates block, a fiscal block, and a debt-structure dynamics block. For example, the macroeconomic block reflects the stylized fact that recessions are associated with both higher funding needs (often financed with Treasury bills) and lower short-term funding costs due to accommodative monetary policy. The model also includes the ability to issue inflation-linked bonds (TIPS) or floating-rate notes (FRNs)[9].

5.2.1 Macroeconomic Block

The macroeconomic block is based on a standard three-equation model: 1. an IS curve linking the unemployment gap (u^*) to its own lags and the stance of monetary policy; 2. a Phillips curve in which inflation depends on its own lags, inflation expectations, and the unemployment gap; and 3. a monetary policy rule in which the stance of policy responds to unemployment and inflation gaps.

These specifications follow the IS and Phillips curve equations in Rudebusch and Williams (2014), where r denotes the federal funds rate, π actual inflation, π^E expected inflation, and R^* the natural rate of interest.

$$u_t^* = 1.57 \cdot u_{t-1}^* - 0.62 \cdot u_{t-2}^* + 0.028 \cdot [0.5 \cdot (r_{t-1} - \pi_{t-1} - R_{t-1}^*) + 0.5 \cdot (r_{t-2} - PI_{t-2} - R_{t-2}^*)] + \varepsilon_{u_t^*}$$

$$\pi_t = 0.58 \cdot \pi_{t-1} + 0.26 \cdot \pi_{t-2} + 0.16 \cdot \pi_t^E - 0.133 \cdot u_{t-1}^* + \varepsilon_{\pi,t}$$

The unemployment gap (u^*) is defined as the deviation of the unemployment rate from its full-employment level. Inflation is measured as the quarterly annualized percent change in the core PCE price index. The standard deviation of shocks is 0.24 for the IS curve and 0.79 for the Phillips curve. The monetary policy block follows the Taylor (1999) 'balanced approach' rule, with an inertial coefficient of 0.85.

$$r_t = 0.85 \cdot r_{t-1} + 0.15 \cdot [R_t^* + \pi_t + 0.5 \cdot (\pi_t - 2) - 2 \cdot u_t^*]$$

R^* is treated as a stochastic variable in our model to capture uncertainty about long-run steady-state outcomes. Specifically, we assume that R^* equals the sum of potential growth and a residual term Z:

[9] See Hartley and Jermann (2024) for further discussion of US Treasury floating rate note mispricing and its implcations for issuance policy as well as Fleckenstein et al. (2014) for the equivalent discussion of inflation-linked-bonds

$$R_t^* = G_t + Z_t$$

G is modeled as a random walk, while Z follows an AR(1) process capturing temporary shocks. The quarterly standard deviation of shocks to G is calibrated at 0.0624, based on the drift in the twenty-year moving average of real GDP growth. The parameters of the Z process are chosen so that its long-run mean is -0.50%, deviations from steady state have a half-life of two years, and the overall volatility of R^* matches that of the twenty-year moving average of the real federal funds rate. Specifically:

$$Z_t = (1-d) \cdot Z_{SS} + d \cdot Z_{t-1} + \varepsilon_{Z,t}$$

where $d = 0.917$, and shocks to the Z process have a standard deviation of 0.018.

5.2.2 Rates Block

In the rates block, we first estimate equations for the 2-year and 10-year term premia using the model of Adrian et al. (2013). Each term premium is regressed on rate volatility, inflation expectations, and the unemployment gap. We then allow for positively correlated shocks across the two term premia equations:

$$TP_t^{10} = -3.59 + 0.207 \cdot u_t^* + 1.22 \cdot \pi_t^E + 1.75 \cdot \sigma_t + \varepsilon_{TP_t^{10}}$$

$$TP_t^2 = -1.32 - 0.014 \cdot u_t^* + 0.477 \cdot \pi_t^E - 0.030 \cdot \sigma_t + 0.420 \cdot TP_t^{10} + \underbrace{\phi_n \cdot D_t}_{\text{supply effects}} + \varepsilon_{TP_t^2}$$

Interest rate volatility, σ, persists at its average level from 2011 to the present. We discipline ϕ_n using high-frequency QRA yield responses from the previous reduced-form empirical analysis.

5.2.3 Fiscal Block

The fiscal block forecasts the primary budget balance B_t as a share of GDP:

$$B_t = 0.34 - 1.50 \cdot u_t^* + \varepsilon_{PRI,t}$$

5.2.4 Debt Structure Dynamics Block

The remaining equations implement the accounting identities for each period: 1. the budget balance is calculated as the sum of the primary balance and the interest cost on the existing stock of debt; 2. the financing need is derived from the total budget balance together with rollovers of maturing debt; and 3. the stock of debt is updated with new issuance, which occurs at market interest rates determined in the rates block.

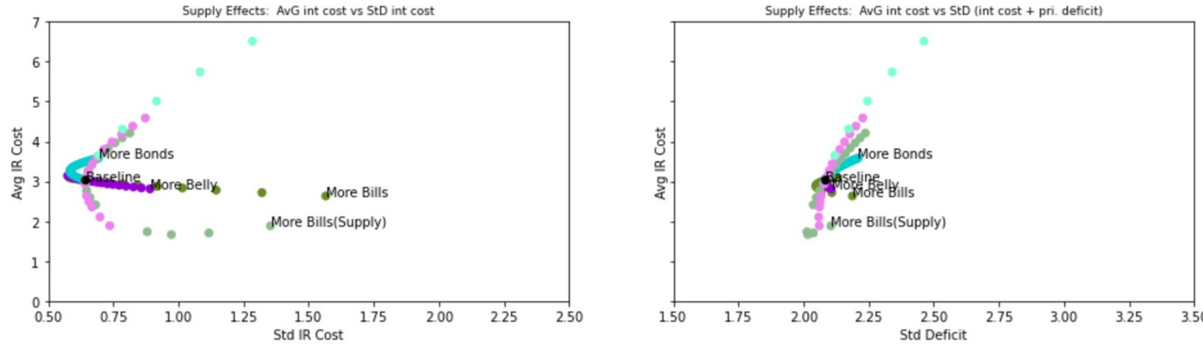
5.3 Debt Management Mean Variance Efficient Frontier and Optimal Issuance

The charts below compare two assumptions about the effect of additional duration on the term spread: a 6 basis point effect per 1 percent of GDP in ten-year equivalents, as in Belton et al. (2018) (Figure 23), versus a 12 basis point effect based on our reduced-form estimates (Figure 22).

Under the empirically-disciplined elasticity, the current U.S. issuance strategy lies interior to the efficient frontier rather than on it, implying that the Treasury is paying a systematic cost premium relative to feasible portfolios with a higher bill share but identical cost volatility.

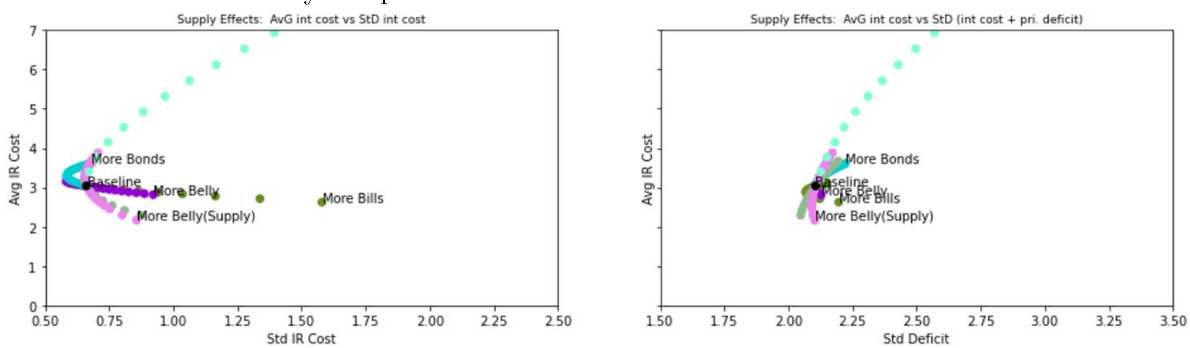
A debt manager with a given risk aversion parameter defining their indifference curves will find a tangency point that allocates a greater share of issuance to bills when using our reduced-form empirical estimates. The results show that when the impact of duration on the term spread is larger, additional short-duration issuance becomes optimal attractive across a wider range of the efficient frontier. Figure 24 shows the optimal issuance by maturity across different levels of debt-manager risk aversion according to the model. Figure 25 shows the change optimal issuance by maturity across different levels of debt-manager risk aversion from current issuance policy.

Figure 22: Mean-Variance government Debt Management Frontier With 12 Basis Point Effect on Term Spread of 1 Percent Additional ten-year equivalent Duration As A Fraction of GDP



Note: Efficient frontier for Treasury debt management when the term spread rises by 12 basis points per 1% of GDP increase in duration supply (empirical estimate). The optimal portfolios feature higher Treasury-bill shares relative to long bonds.

Figure 23: Mean-Variance government Debt Management Frontier With 6 Basis Point Effect on Term Spread of 1 Percent Additional ten-year equivalent Duration As A Fraction of GDP



Note: Efficient frontier using a 6 basis-point elasticity (Belton et al. (2018) calibration). Comparing to the case with a 12 basis point elasticity suggests stronger duration effects make short-term issuance more attractive along the efficient frontier.

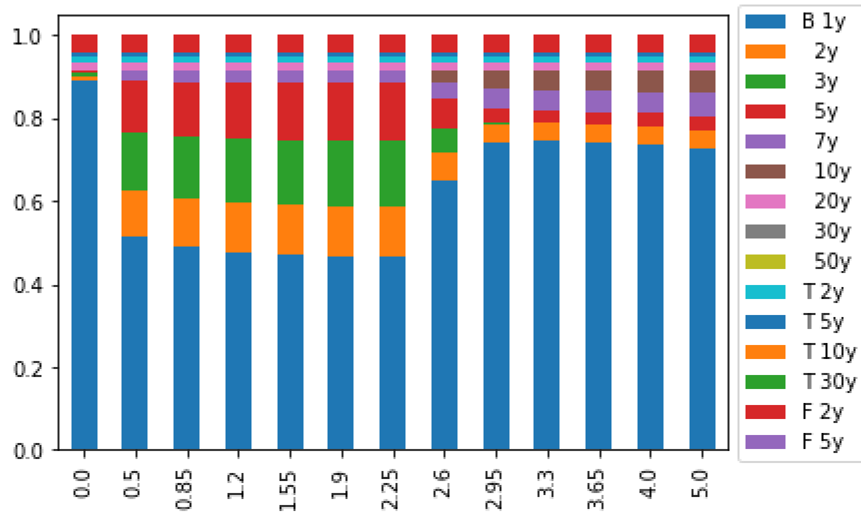
One can use the later years of the model's simulation (year 19 of the 20 year simulation) to quantify the effects of debt management on interest costs and the effects of various parameters.

Let's suppose that the debt manager were to issue all new debt as 1-year nominal bonds. With no supply effects in the model, the annual average interest expense associated with issuing all new debt at 1-year duration is $2.0 trillion. This is compared to $1.8 trillion in the presence of a 6 basis point term-spread supply effect of 1% of additional Ten Year Equivalent Duration. Hence, our reduced-form empirical estimates, which find that term-spread sensitivity to longer-duration debt is higher than previously estimated, suggests that issuing lower-duration debt would lead to a lower average interest expense than previously thought.

This is a simple illustration of how our high-frequency empirical findings overall point to issuing lower duration debt as being less costly to the government.

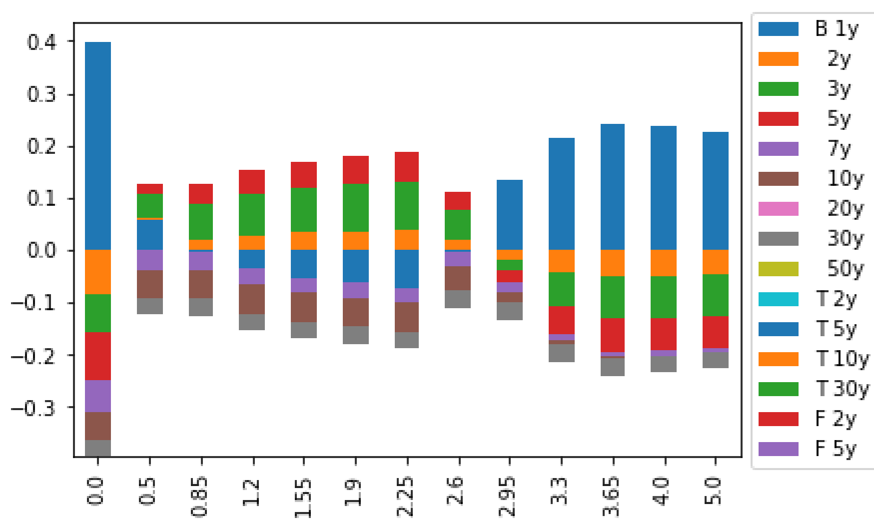
To our knowledge, this is the first paper to embed high-frequency causal estimates of Treasury supply effects into an operational debt management model used by policymakers, linking event-study elasticities directly to optimal issuance strategy. The following section provides a microfoundation for these duration effects using an overlapping-generations model where limited risk-sharing generates a positive term premium.

Figure 24: Optimal Treasury Issuance by Maturity Across Risk-Aversion Levels



Note: For each level of debt-manager risk aversion, the mean-variance frontier tangency portfolio implies a higher share of bill financing when the term-spread elasticity equals 12bp per 1% of GDP in duration supply.

Figure 25: Change in Optimal Issuance Relative to Current Treasury Policy



Note: Bars show the difference between model-implied optimal issuance and the current U.S. maturity profile. Higher duration sensitivity shifts issuance away from long-term bonds and into bills across nearly all risk-aversion levels.

6 Conclusion

This paper uses U.S. Treasury Quarterly Refunding Announcements (QRAs) as exogenous variation in bond supply information to study how government debt management policy affects borrowing costs. We combine high-frequency data on market reactions to QRAs with new measures of pre-announcement expectations from financial analyst reports to estimate the effect of supply surprises on yields.

A key implication of our maturity-specific results is that Treasury yields do not respond to supply shocks solely through aggregate duration. Instead, issuance at short and intermediate maturities exerts disproportionate influence on benchmark yields, while long-end issuance plays little independent role once nearer maturities are accounted for. This evidence favors preferred-habitat models with segmented demand over frictionless theories in which all duration is priced uniformly.

Our maturity-specific results show that short- and intermediate-maturity issuance shocks propagate along the yield curve, while long-end issuance does not independently affect intermediate yields, rejecting a single-factor aggregate-duration view and instead supporting preferred-habitat models with imperfect arbitrage and money-like short-term debt. This segmentation implies that shifting issuance toward bills and the belly can lower borrowing costs without proportionally increasing long-term yields, providing a micro-founded rationale for active maturity management.

In contrast to prior work, we estimate relative Treasury supply effects using intraday minutely Treasury yield data (within high-frequency windows around QRAs) together with high-frequency duration (issuance) surprises estimated from announced issuance data and expected issuance data from analyst forecasts.

We find that an unexpected announcement of an issuance within the next 3 months with a ten-year duration equivalent equal to 1% of GDP causes a 0.07% to 0.20% increase in the 3-month to 10-year term spread. In magnitudes, these results suggest that the term spread is more sensitive to the duration of the term structure than past estimates in the literature (Greenwood and Vayanos (2014), Belton et al. (2018), Wright (2022)). The effects are concentrated in longer maturities while short-term yields are largely unaffected by duration shocks.

We derive the quantitative implications of our documented empirical effects in an extended mean–variance model of optimal debt management that the Treasury Borrowing Advisory Committee uses to advise the Treasury Department (Belton et al. (2018)). We find that minimizing government interest costs implies that the optimal weighted-average maturity of government debt should be shorter than it has been over recent decades.

QRA announcements about issuance over the next quarter likely signal Treasury's intended issuance path well beyond the three-month horizon, given the infrequency with which issuance glide paths are revised. As a result, the true surprises in duration may be larger than what we measure, which may bias upward our estimated elasticities.

Directionally, our results are consistent with simple theories by which term premia adjust to the duration (weighted average time to receive cash flows) of the outstanding supply of debt. We formalize the following intuition in a novel OLG model: when agents are young, they can buy either short-term debt or long-term debt. When they are old, the short-term bonds mature and the long-term bonds are forced be sold. Imperfect risk sharing and finite horizons creates risk premia for long-term bonds.

Our overlapping-generations framework differs from the canonical models of (Barro, 1974), Bohn (1990), Angeletos (2002), and (Faraglia et al., 2019). Whereas those focus respectively on Ricardian equivalence, exogenous interest rates, full Arrow–Debreu spanning, or issuance frictions, our model highlights how finite horizons and incomplete intergenerational risk-sharing endogenously generate duration risk premia that align with the high-frequency empirical evidence on Treasury maturity shocks.

Extended versions of such simple models that include intermediaries (Vayanos and Vila (2021)) explain that much of this effect may be compensation to financial intermediaries for exposing their capital to the risk of inventories of Treasuries. The empirical patterns we document are also consistent with stories around the demand for money-like assets, in line with models such as Greenwood et al. (2015), where short-term bills provide monetary services.

Our findings speak to current policy debates. The Treasury's 2023 shift toward greater reliance on short-term bills raises questions about whether this offsets, at least in part, the Federal Reserve's quantitative tightening and sales of Treasuries from its balance sheet, as suggested by Miran and Roubini (2024). If so, this would have implications for coordination (or lack thereof) between fiscal and monetary authorities. Ultimately, however, the mandate of the Treasury's Office of Debt Management is to minimize financing costs over time, while the Federal Reserve should take debt management policy as given when setting balance sheet policy. Exploring these fiscal–monetary interactions further remains an important topic for future research.[10]

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A Appendix A: Extra Figures

Figure A1: U.S. Treasury Quarterly Refunding Announcement Standard Text From The November 1, 2023 (Fourth Quarter 2023) Announcement

PRESS RELEASES

Quarterly Refunding Statement of Assistant Secretary for Financial Markets Josh Frost

November 1, 2023

WASHINGTON — The U.S. Department of the Treasury is offering $112 billion of Treasury securities to refund approximately $102.2 billion of privately-held Treasury notes maturing on November 15, 2023. This issuance will raise new cash from private investors of approximately $9.8 billion. The securities are:

- A 3-year note in the amount of $48 billion, maturing November 15, 2026;
- A 10-year note in the amount of $40 billion, maturing November 15, 2033; and
- A 30-year bond in the amount of $24 billion, maturing November 15, 2053.

The 3-year note will be auctioned at 1:00 p.m. ET on Tuesday, November 7, 2023. The 10-year note will be auctioned at 1:00 p.m. ET on Wednesday, November 8, 2023. The 30-year bond will be auctioned at 1:00 p.m. ET on Thursday, November 9, 2023. All of these auctions will take place on a yield basis and will settle on Wednesday, November 15, 2023.

The balance of Treasury financing requirements over the quarter will be met with regular weekly bill auctions, cash management bills (CMBs), and monthly note, bond, Treasury Inflation-Protected Securities (TIPS), and 2-year Floating Rate Note (FRN) auctions.

Source: https://home.treasury.gov/news/press-releases/jy1864

Figure A2: U.S. Treasury Quarterly Refunding Announcement Statement One Quarter Ahead Issuance Table From The November 1, 2023 (Fourth Quarter 2023) Announcement

	2-Year	3-Year	5-Year	7-Year	10-Year	20-Year	30-Year	FRN
Aug-23	45	42	46	36	38	16	23	24
Sep-23	48	44	49	37	35	13	20	24
Oct-23	51	46	52	38	35	13	20	26
Nov-23	**54**	**48**	**55**	**39**	**40**	**16**	**24**	**26**
Dec-23	**57**	**50**	**58**	**40**	**37**	**13**	**21**	**26**
Jan-24	**60**	**52**	**61**	**41**	**37**	**13**	**21**	**28**

Source: https://home.treasury.gov/news/press-releases/jy1864

Figure A3: J.P. Morgan Analyst Report Quarterly Refunding Issuance Size Expectations Ahead of Fourth Quarter 1995 U.S. Treasury Quarterly Refunding Announcement

New York
October 27, 1995

Morgan Guaranty Trust Company
Economic Research
Marc Wanshel (1-212) 648-3032
wanshel_m@jpmorgan.com

5-year notes settle on October 31. A $500 million allotment to foreign central banks was tacked onto the size of the 2-year note auctioned Tuesday, despite the previous week's statement that there would be no such "add-ons" to the note issues. Although the add-on was said to have been caused by an oversight, the error would not likely have been allowed to stand unless Treasury thought it would have some room to maneuver on and after October 31.

Tentative auction schedule announced

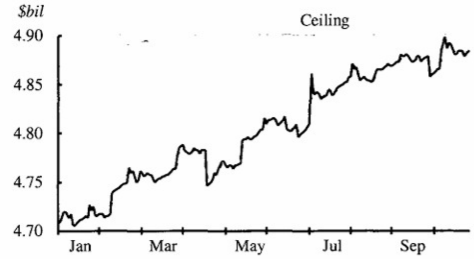
On Wednesday, the Treasury released a tentative schedule of auction dates for November which seemed intended to mitigate some of the uncertainties resulting from the debt limit. The schedule (which provides announcement, auction, and settlement dates, but not the sizes of offerings) confirms that the government intends to issue a cash management bill on November 3. The Treasury tentatively scheduled the monthly 2- and 5-year note auctions for November 28 and 29 – a week later than usual, probably to avoid the risk of congestion in the holiday-shortened week of November 20 if the debt limit impasse leads to a delay in the refunding auctions.

Assuming the offering size of the weekly bills announced next Tuesday and issued November 9 closely matches maturities, as did this week's offering, a cash management bill of $10 billion $15 billion should prove adequate to meet the November 3 social security payment and maintain a prudent $5 billion cash balance through November 14. It is very likely to carry the same January 25 maturity as the undersized 3-month bill that was auctioned this week, bringing the issue to above-normal size.

What to expect in financing announcement...

The Treasury is scheduled to announce its quarterly market borrowing estimates for the fourth and first quarters on October 30. The government's debt managers often use the financing announcement (or the refunding announcement two days later) to discuss intermediate- and long-term issues in Treasury finance. At this week's briefing, interest will center on any insights to Treasury's strategy for meeting sharply higher note and bond maturities in 1996 – particularly the possibility of an increase in the frequency of 10-year offerings and the introduction of index-linked or floating rate notes or bonds. But Treasury does not yet appear close to any decisions on altering the current issue cycle, and is likely to say only that it is continuing to study the feasibility of new offerings.

Total debt subject to limit



The borrowing estimate for the fourth quarter is likely to be close to the preliminary estimate made in July – $60 billion to $65 billion to reach an ending cash balance of $20 billion. The preliminary figure for the first quarter of next year is expected to be in the range of $70 billion to $80 billion, also to reach an ending cash balance of $20 billion. (Actual government borrowing in Q1 is likely to be $10 billion lower.)

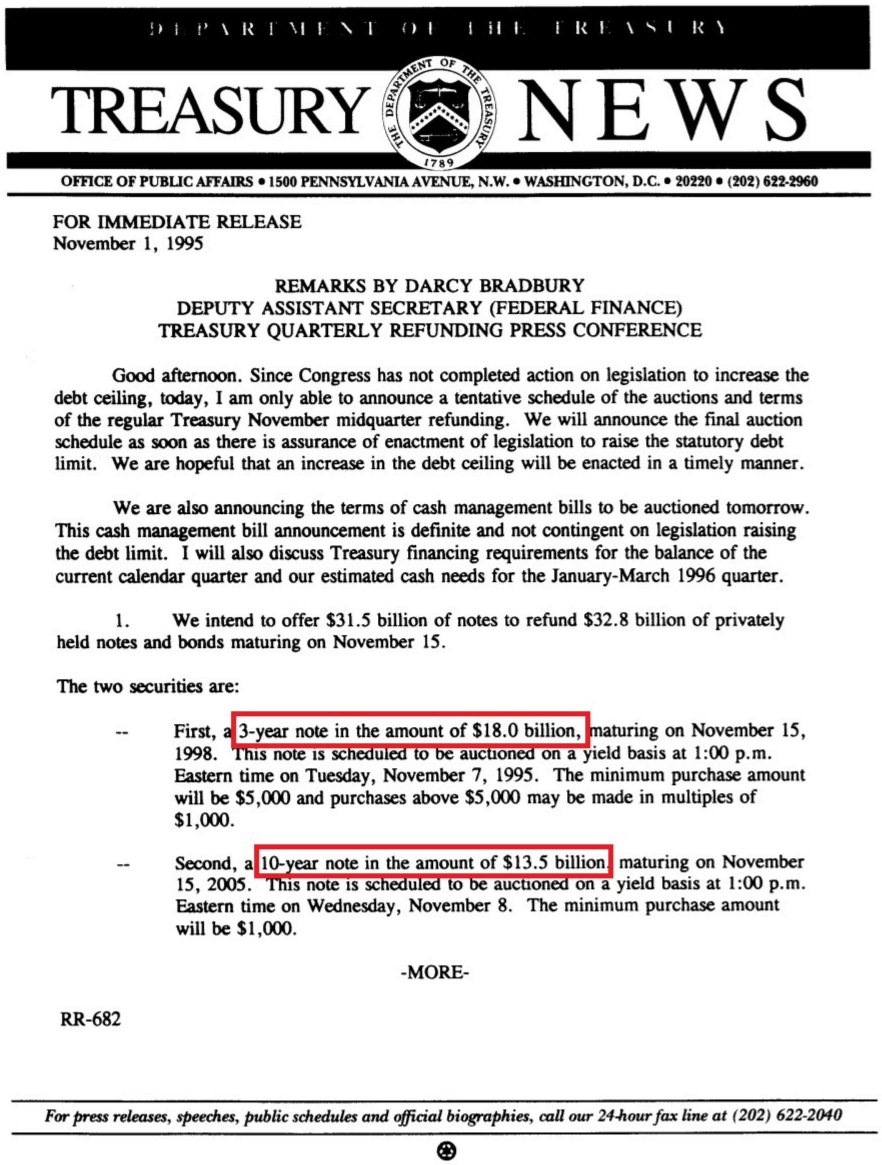
...and in auction announcements

In addition to the October 30 financing announcement, the Treasury is tentatively scheduled to announce on November 1 its refunding offering of 3- and 10-year notes and a cash management bill. At their expected offering sizes of $18.5 billion and $13.5 billion, respectively, the 3- and 10-year notes do not pose a debt limit problem of themselves – they would produce a net paydown $0.8 billion. However, the November 15 issue date poses a clear legal problem in the absence of an increase in the debt limit (Treasury will have insufficient cash to make $22 billion of interest payments on that date without new borrowing that would bring its debt above the existing limit). The refunding announcement will probably retain the tentative auction dates (November 7 and 8) indicated yesterday, but the auctions will almost surely be made conditional on passage of an increase in the debt limit.

The probability that the auctions will go off on time has risen over the past week – ironically due to the slower-than-anticipated progress of budget legislation through Congress. Since it is unlikely that Congress will have a bill ready by the November 15 "drop dead" date, the Republican leadership, at least, is more likely to support a temporary increase in the debt limit soon enough to avoid disruptions to the auctions.

Source: J.P. Morgan Markets

Figure A4: 1995 U.S. Treasury Quarterly Refunding Announcement



DEPARTMENT OF THE TREASURY

TREASURY NEWS

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FOR IMMEDIATE RELEASE
November 1, 1995

REMARKS BY DARCY BRADBURY
DEPUTY ASSISTANT SECRETARY (FEDERAL FINANCE)
TREASURY QUARTERLY REFUNDING PRESS CONFERENCE

Good afternoon. Since Congress has not completed action on legislation to increase the debt ceiling, today, I am only able to announce a tentative schedule of the auctions and terms of the regular Treasury November midquarter refunding. We will announce the final auction schedule as soon as there is assurance of enactment of legislation to raise the statutory debt limit. We are hopeful that an increase in the debt ceiling will be enacted in a timely manner.

We are also announcing the terms of cash management bills to be auctioned tomorrow. This cash management bill announcement is definite and not contingent on legislation raising the debt limit. I will also discuss Treasury financing requirements for the balance of the current calendar quarter and our estimated cash needs for the January-March 1996 quarter.

1. We intend to offer $31.5 billion of notes to refund $32.8 billion of privately held notes and bonds maturing on November 15.

The two securities are:

-- First, a 3-year note in the amount of $18.0 billion, maturing on November 15, 1998. This note is scheduled to be auctioned on a yield basis at 1:00 p.m. Eastern time on Tuesday, November 7, 1995. The minimum purchase amount will be $5,000 and purchases above $5,000 may be made in multiples of $1,000.

-- Second, a 10-year note in the amount of $13.5 billion, maturing on November 15, 2005. This note is scheduled to be auctioned on a yield basis at 1:00 p.m. Eastern time on Wednesday, November 8. The minimum purchase amount will be $1,000.

-MORE-

RR-682

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Source: U.S. Treasury

Figure A5: J.P. Morgan Analyst Report Quarterly Refunding Issuance Size Expectations Ahead of Fourth Quarter 2023 U.S. Treasury Quarterly Refunding Announcement

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adding to the need to increase borrowing capacity.

Accordingly, we expect Treasury to essentially replicate last quarter's increases at next week's announcement (**Figure 4**). In particular, we expect Treasury to increase the 2- and 5-year tenors by $3bn per month, the 3-year tenor by $2bn per month, and the 7-year by $1bn per month. In addition, we expect both the new issue and reopening auction sizes of the 10-year note to be raised by $3bn, the 20-year bond by $1bn, and the 30-year bond by $2bn. Further, Treasury should continue to make incremental increases to TIPS auction sizes in order to maintain a stable share of TIPS as a percentage of total marketable debt outstanding: we expect Treasury to increase the January 10-year TIPS new auction size by $1bn. Lastly, we expect Treasury will increase 2-year FRN auction sizes by an additional $2bn in January. At the August meeting, a TBAC member presented two scenarios: one, a "neutral issuance" scenario, where coupon issuance is increased proportionately along the curve, and another where lower term premium supports increased issuance in longer tenors. Ultimately, the way Treasury decided to proceed is largely consistent with the first scenario. Further, following this presentation, "There was a robust discussion on the different assumptions used in the model, particularly with regard to term premia, and how those assumptions may influence the Model's conclusions."[1] Treasury ultimately went forward with the bulk of coupon auction increases in the intermediate sector, even before the recent rise in term premium, which makes us feel comfortable that the distribution of coupon auction increases should be identical to what was announced in August.

Figure 4: We continue to expect Treasury to increase auction sizes through the middle of next year

Realized and projected gross issuance sizes for coupon Treasuries for last quarter of 2023 and full year 2024, reopenings shaded in grey; $bn

	2s	3s	5s	7s	10s	20s	30s	5y TIPS	10y TIPS	30y TIPS	2y FRN	Total
Oct 23	51	46	52	38	35	13	20	22			26	303
Nov 23	54	48	55	39	41	17	25		15		24	318
Dec 23	57	50	58	40	38	14	22	20			24	323
2023 Total	**549**	**510**	**561**	**435**	**417**	**163**	**242**	**82**	**94**	**17**	**282**	**3352**
Jan 24	60	52	61	41	38	14	22		18		28	334
Feb 24	63	54	64	42	44	18	27			10	26	348
Mar 24	66	56	67	43	41	15	24		16		26	354
Apr 24	69	58	70	44	41	15	24	23			30	374
May 24	72	60	73	45	47	19	29		16		28	389
Jun 24	75	62	76	46	44	16	26	21			28	394
Jul 24	78	64	79	47	44	16	26		19		32	405
Aug 24	78	64	79	47	47	19	29			9	30	402
Sep 24	78	64	79	47	44	16	26		17		30	401
Oct 24	78	64	79	47	44	16	26	23			32	409
Nov 24	78	64	79	47	47	19	29		17		30	410
Dec 24	78	64	79	47	44	16	26	21			30	405
2024 Total	**873**	**726**	**885**	**543**	**525**	**199**	**314**	**88**	**103**	**19**	**350**	**4625**

Source: US Treasury, J.P. Morgan

On balance, we expect $3,034bn in net privately-held borrowing in 2023 and $2,432bn in 2024. T-bills should account for nearly two-thirds of this year's net issuance, but less

1. https://home.treasury.gov/news/press-releases/jy1669

Source: J.P. Morgan Markets

Figure A6: Bloomberg Aggregated Fourth Quarter Primary Dealer One Quarter Ahead Expectations Published on October 31, 2023 Ahead of November 1, 2023 U.S. Treasury Quarterly Refunding Announcement

	2Y(51)	3Y (46)	5Y (52)	7Y (38)	10Y(38/35/35)	20Y(16/13/13)	30Y(23/20/20)
Bank of Montreal	54/57/60	48/50/52	55/58/61	39/40/41	41/38/38	17/14/14	25/22/22
Bank of Nova Scotia	54/57/60	48/50/52	55/58/61	39/40/41	41/38/38	17/14/14	25/22/22
BNP Paribas	54/57/60	48/50/52	55/58/61	39/40/41	40/37/37	17/14/14	24/21/21
Barclays	54/57/60	48/50/52	54/56/58	39/40/41	40/37/37	16/13/13	24/21/21
Bank of America	54/57/60	48/50/52	55/58/61	39/40/41	41/38/38	17/14/14	25/22/22
Cantor Fitzgerald	54/57/60	48/50/52	55/58/61	39/40/41	41/38/38	17/14/14	25/22/22
Citigroup	54/57/60	48/50/52	55/58/61	40/42/44	41/38/38	17/14/14	26/23/23
Daiwa	52/53/56	46/46/50	53/54/56	39/40/43	39/36/36	16/13/13	23/20/20
Deutsche Bank	54/57/60	48/50/52	55/58/61	39/40/41	40/37/37	16/13/13	24/21/21
Goldman Sachs	53/55/57	48/50/52	54/56/58	39/40/41	40/37/37	17/14/14	24/21/21
HSBC	54/57/60	48/50/52	55/58/61	39/40/41	41/38/38	17/14/14	25/22/22
Jefferies	53/55/57	47/48/49	53/54/55	39/40/41	40/37/37	17/14/14	25/22/22
JPMorgan	54/57/60	48/50/52	55/58/61	39/40/41	41/38/38	17/14/14	25/22/22
Mizuho	54/57/60	48/50/52	55/58/61	40/42/44	41/38/38	17/14/14	25/22/22
Morgan Stanley	53/55/57	48/50/52	54/56/58	39/40/41	40/37/37	16/13/13	24/21/21
NatWest	54/57/60	48/50/52	55/58/61	39/40/41	41/38/38	17/14/14	25/22/22
Nomura	54/57/60	48/50/52	55/58/61	39/40/41	41/38/38	17/14/14	25/22/22
RBC	54/57/60	48/50/52	55/58/61	39/40/41	41/38/38	17/14/14	25/22/22
Santander	54/57/60	49/52/55	55/58/61	40/42/44	41/38/38	17/14/14	25/22/22
Societe Generale	54/56/58	48/50/52	55/58/60	39/40/41	40/37/37	16/13/13	24/21/21
TD	54/57/60	48/50/52	55/58/61	39/40/41	41/38/38	17/14/14	25/22/22
UBS	54/57/60	48/50/52	55/58/61	39/40/41	41/38/38	17/14/14	25/22/22
Wells Fargo	53/55/57	48/50/52	54/56/58	39/40/41	40/37/37	17/14/14	24/21/21

Source: Bloomberg(https://www.bloomberg.com/news/articles/2023-10-31/wall-street-predicts-record-114-billion-treasury-quarterly-debt-refunding). Here x/y/z is the Nov/Dec/Jan estimate by dealer. Bracket terms are the previous quarters issuance.

Figure A7: J.P. Morgan Analyst Report Treasury Quarterly Refunding Forecast Surprise Versus Bloomberg Survery Median Forecast Surprise

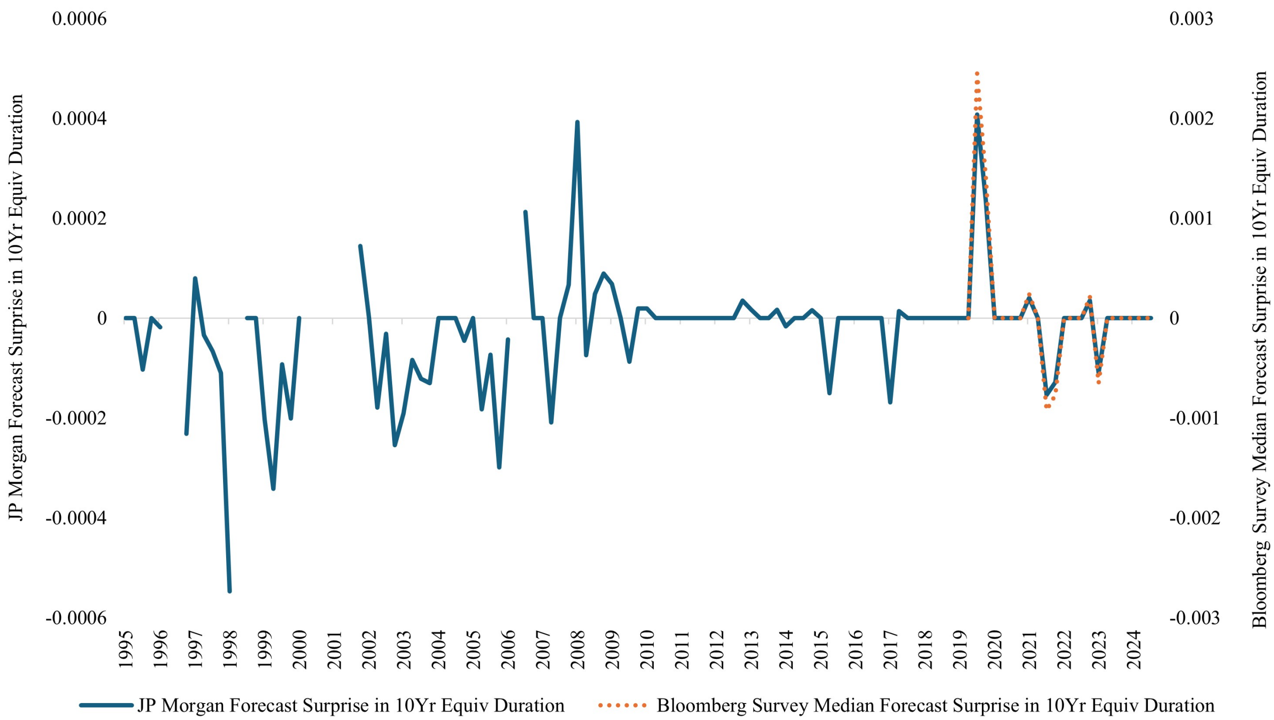


Table 5: U.S. Treasury Quarterly Refunding high-frequency Yield Impacts Versus Unexpected One Quarter Ahead Shift in Treasury Duration Using Only Median Bloomberg Survey Surprise Data

	U.S. Treasury Quarterly Refunding		
	high-frequency (30-minute Window) Change In Yield (%)		
	10-year yield minus 3-month yield	10-year yield	3-month yield
DurationShock	15.151**	10.965***	-4.396
	(3.049)	(2.798)	(1.303)
constant	-0.003	-0.002	0.001
	(0.005)	(0.004)	(0.002)
R-Square	0.469	0.374	0.305
N	22	22	22

Notes: * $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$

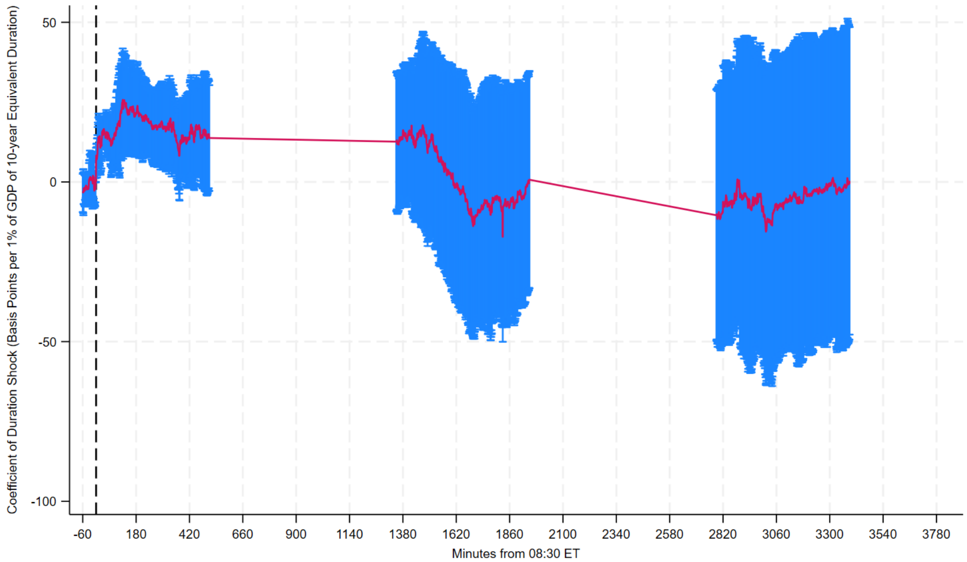
Notes: Duration is measured in one quarter ahead Treasury Ten-Year Equivalents as a Fraction of GDP. Standard errors are Newey–West heteroscedasticity robust standard errors.

Table 6: High-Frequency Yield Responses to QRA Duration Shocks Controlling for GDP Surprises

	U.S. Treasury Quarterly Refunding		
	High-Frequency (30-minute) Change in Yield/Spread (%)		
	10Y Treasury Yield	Term Spread (10Y–3M)	3M Treasury Yield
Duration Shock	8.235***	7.444***	0.790
	(2.151)	(2.572)	(1.371)
GDP Surprise	0.003	-0.002	0.005
	(0.006)	(0.007)	(0.004)
constant	0.001	0.018	-0.000
	(0.001)	(0.014)	(0.001)
R-Square	0.177	0.110	0.024
N	72	72	72

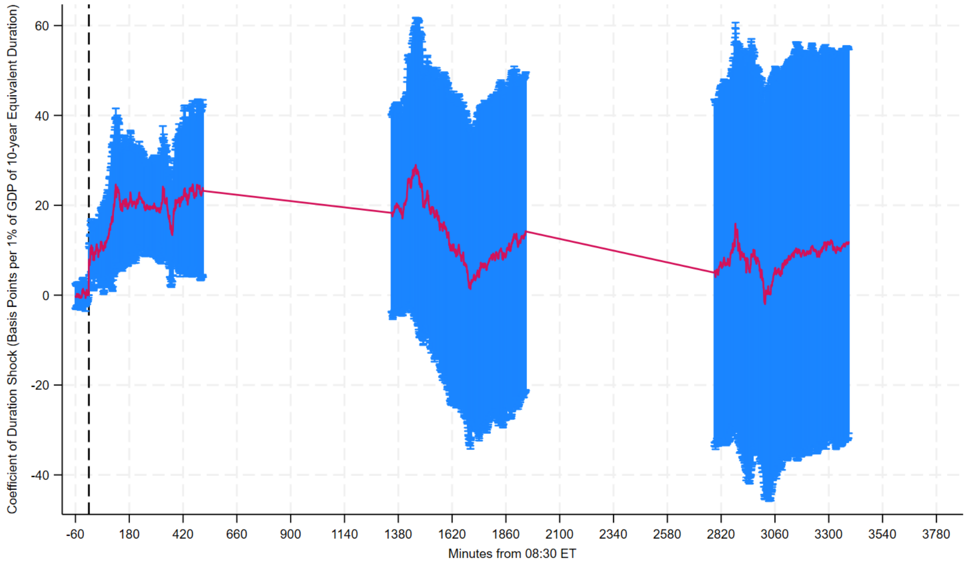
Notes: * $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$. Duration is measured as one-quarter-ahead Treasury ten-year equivalents as a fraction of GDP. Standard errors are Newey–West heteroscedasticity robust.

Figure A8: 3-Day Horizon Study Plot of 10-Year Treasury Yield Minus 3-Month Treasury Yield Cumulative Market Reaction To 1% of GDP Surprise in 10-Year Equivalent Duration At U.S. Treasury Quarterly Refunding Announcement



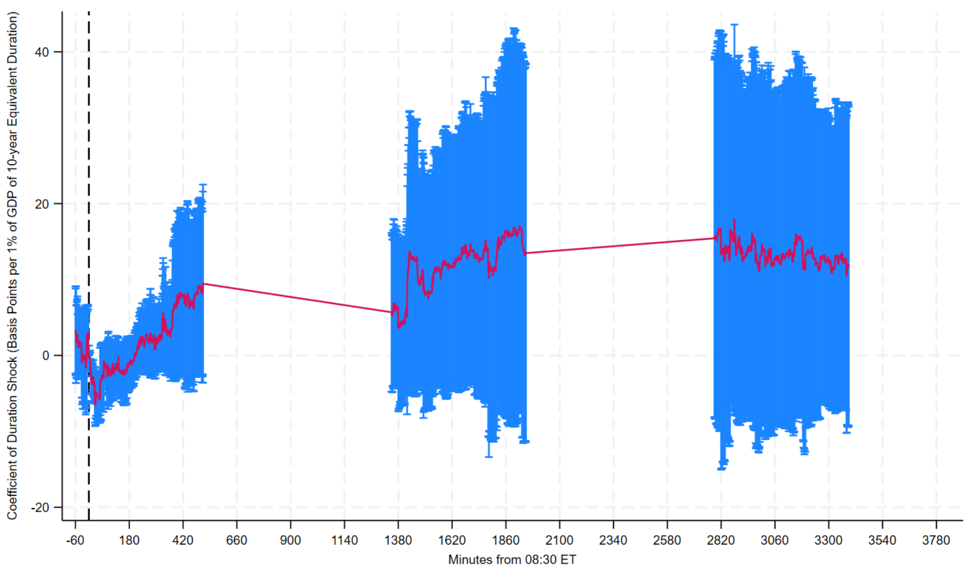
Notes: Daily aggregated event study plot showing the cumulative market reaction of the 10-year minus 3-month Treasury yield term spread to a 1% of GDP surprise in 10-year equivalent duration at U.S. Treasury Quarterly Refunding Announcements over a 3-day horizon. The red line represents coefficient estimates from regressions of yield changes (relative to pre-announcement levels at 8:29 AM ET) on duration shocks, aggregated to daily frequency. The blue shaded area shows 95% confidence intervals based on cluster-robust standard errors (clustered by event). The vertical dashed line at zero marks the announcement time.

Figure A9: 3-Day Horizon Daily Event Study Plot of 10-Year Treasury Yield Cumulative Market Reaction To 1% of GDP Surprise in 10-Year Equivalent Duration At U.S. Treasury Quarterly Refunding Announcement



Notes: Daily aggregated event study plot showing the cumulative market reaction of the 10-year Treasury yield to a 1% of GDP surprise in 10-year equivalent duration at U.S. Treasury Quarterly Refunding Announcements over a 3-day horizon. The red line represents coefficient estimates from regressions of yield changes (relative to pre-announcement levels at 8:29 AM ET) on duration shocks, aggregated to daily frequency. The blue shaded area shows 95% confidence intervals based on cluster-robust standard errors (clustered by event). The vertical dashed line at zero marks the announcement time.

Figure A10: 3-Day Horizon Event Study Plot of 3-Month Treasury Yield Market Reactions To Quarterly U.S. Treasury Refunding Announcement Versus Deviation Treasury Issuance Expectations in ten-year equivalent Duration



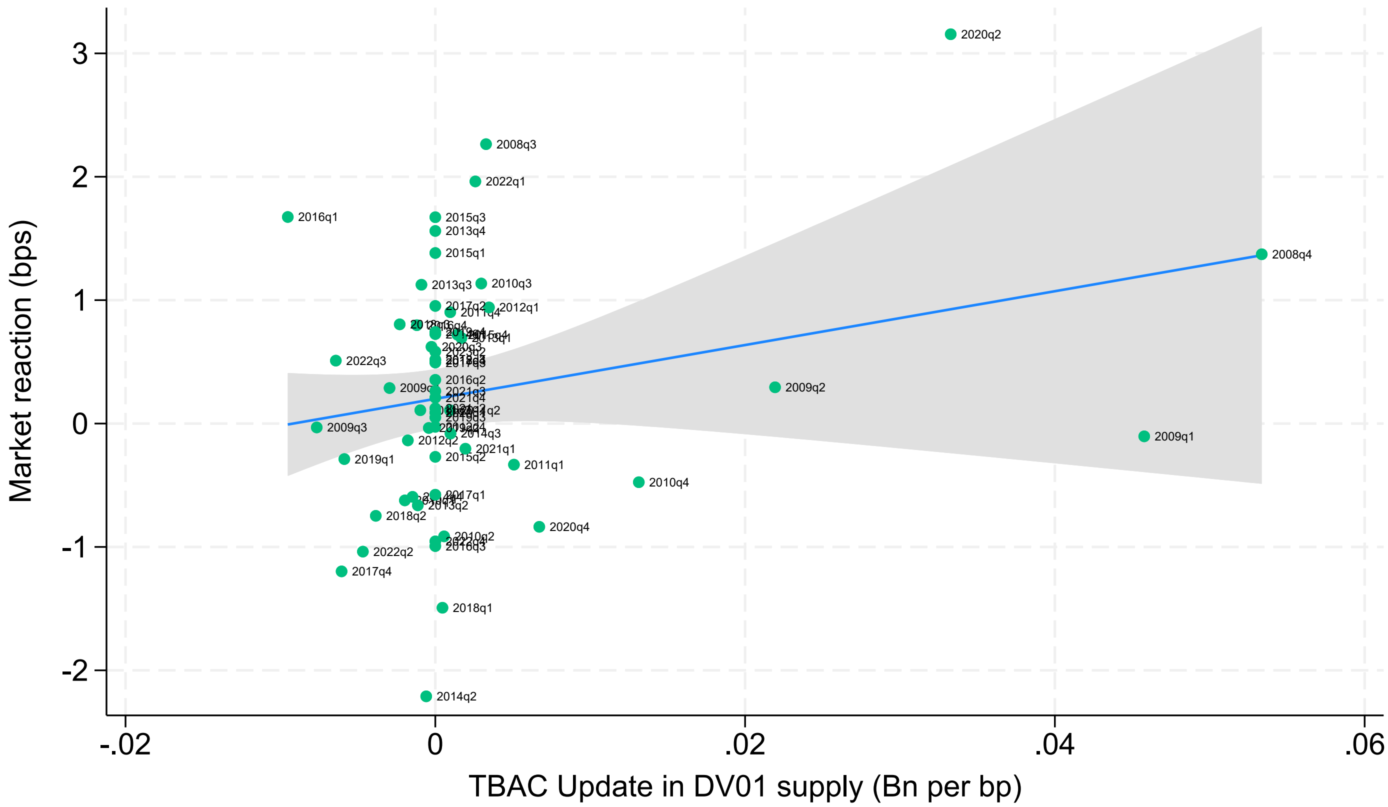
Notes: Daily aggregated event study plot showing the cumulative market reaction of the 3-month Treasury yield to a 1% of GDP surprise in 10-year equivalent duration at U.S. Treasury Quarterly Refunding Announcements. The red line represents coefficient estimates from regressions of yield changes (relative to pre-announcement levels at 8:29 AM ET) on duration shocks, aggregated to daily frequency. The blue shaded area shows 95% confidence intervals based on cluster-robust standard errors (clustered by event). The vertical dashed line at zero marks the announcement time.

Table 7: Maturity-Specific Issuance Shocks and Local Yield Responses (Alternative Sample Excluding Fourth Quarter 2008 QRA)

	1m	3m	6m	12m	2y	3y	5y	7y	10y	20y	30y
3-Year Issuance Shock	−0.00231	−0.00565	0.00172	0.00225	0.00268	0.00627	0.00924**	0.00909*	0.00840	−0.00412	0.00698
	(0.00196)	(0.00439)	(0.00197)	(0.00273)	(0.00361)	(0.00429)	(0.00456)	(0.00484)	(0.00526)	(0.00886)	(0.00641)
10-Year Issuance Shock	−0.00134	−0.00163	−0.00163**	0.00114	0.00191	0.00306	0.00308	0.00333	0.00342	0.00294	0.00336
	(0.00117)	(0.00117)	(0.00066)	(0.00221)	(0.00310)	(0.00289)	(0.00256)	(0.00224)	(0.00218)	(0.00326)	(0.00297)
30-Year Issuance Shock	0.00077	−0.00047	0.00059	−0.00125	−0.00176	−0.00212	−0.00122	0.00045	0.00182	0.00279	0.00397
	(0.00108)	(0.00101)	(0.00084)	(0.00188)	(0.00235)	(0.00221)	(0.00198)	(0.00193)	(0.00208)	(0.00418)	(0.00289)
Observations	74	74	74	74	74	74	74	74	74	44	74
R^2	0.024	0.145	0.031	0.015	0.018	0.038	0.051	0.083	0.120	0.043	0.154

Notes: The table reports event-level regressions of high-frequency changes in Treasury yields around U.S. Treasury Quarterly Refunding Announcements on issuance surprises at the 3-year, 10-year, and 30-year maturities. Columns correspond to Treasury-bill–relative yield measures at the indicated maturities. Issuance surprises are scaled into comparable risk units. Heteroskedasticity-robust standard errors are reported in parentheses. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.10$.

Figure A11: 10-Year Treasury Yield Market Reactions To Quarterly U.S. Treasury Refunding Announcement Versus TBAC Issuance Table Updates



Notes: Such market reactions are slightly correlated with the TBAC update. Figure 3 and Figure 4 shows that the high-frequency changes in the 10-year Treasury yield is slightly positively correlated with the TBAC update in DV01 supply (duration) while the 3-month Treasury yield is largely uncorrelated with the TBAC update in DV01 supply (duration).

Figure A12: 3-Month Treasury Bill Yield Market Reactions To Quarterly U.S. Treasury Refunding Announcement Versus TBAC Issuance Table Updates

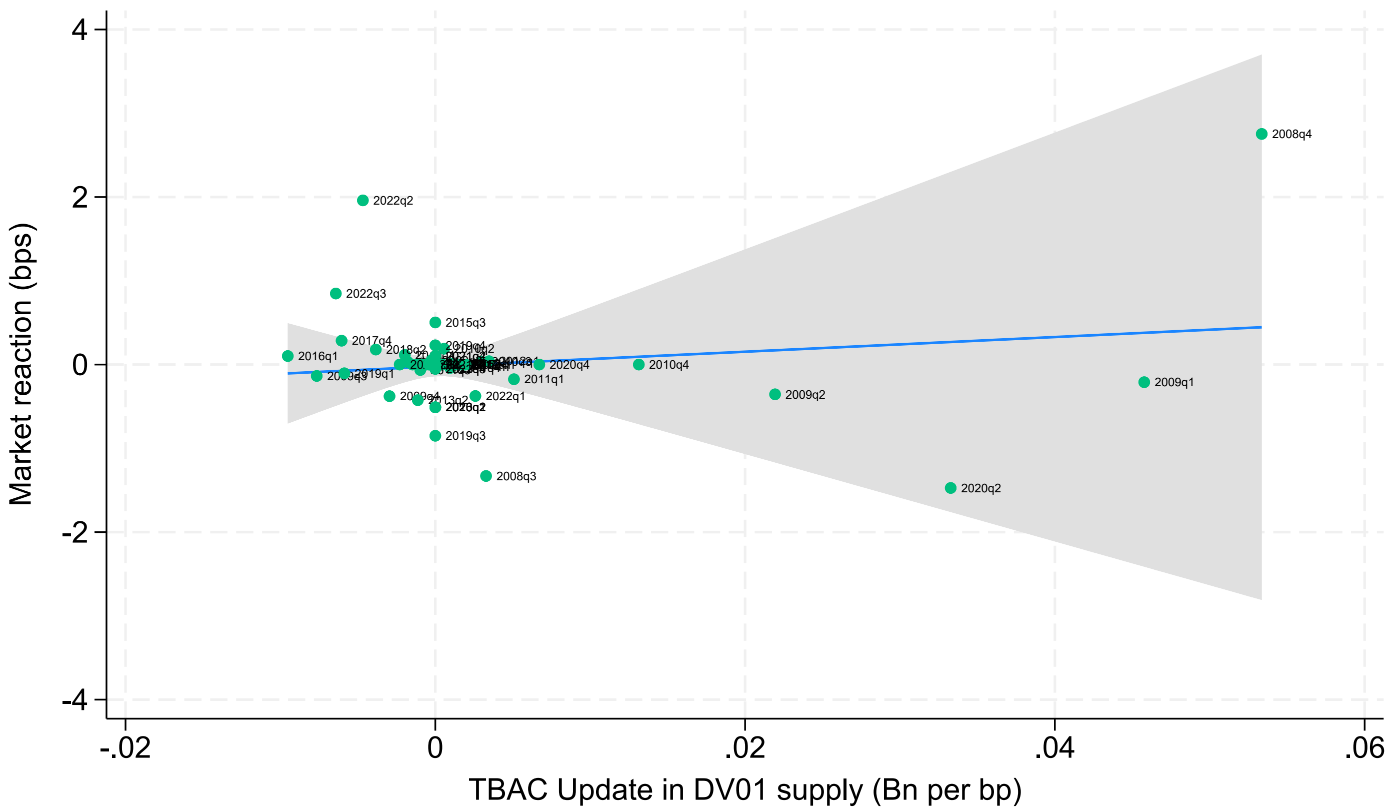


Figure A13: Offered Treasury Bills

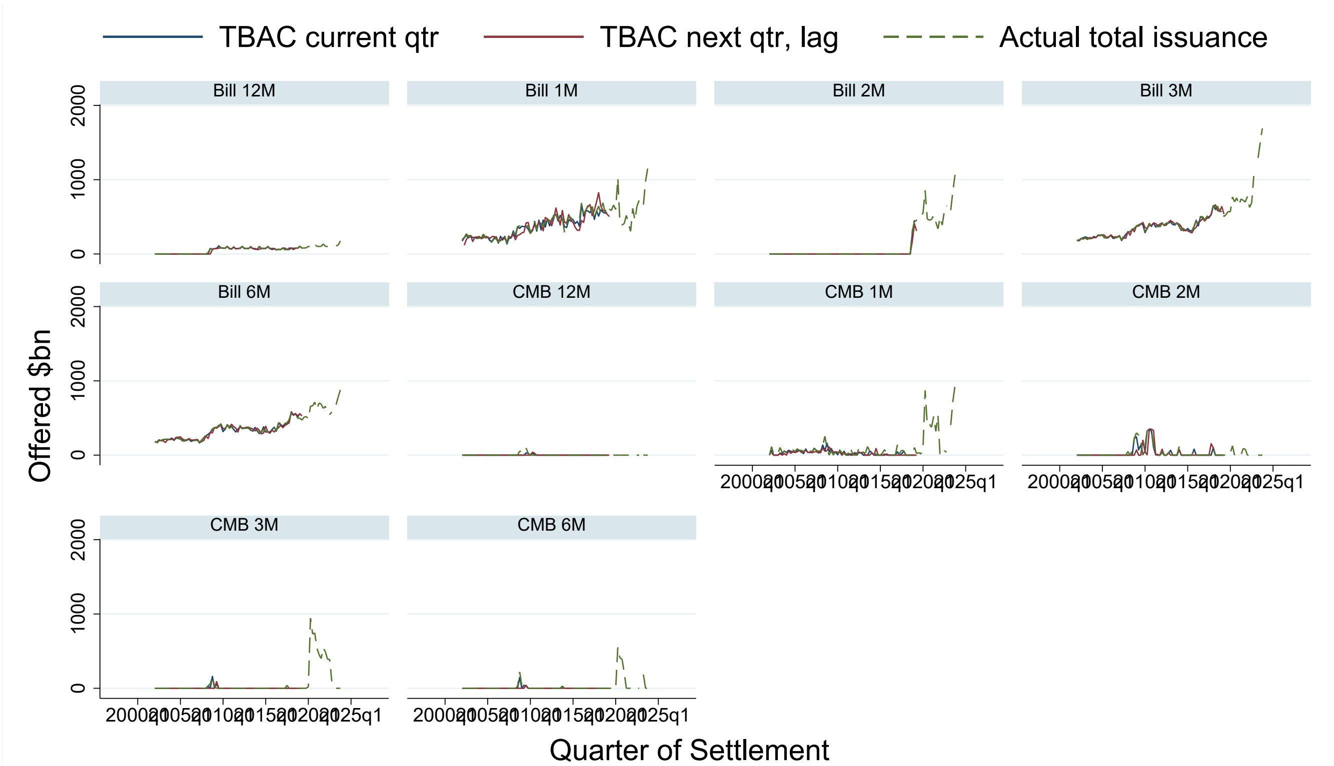


Figure A14: Offered Treasury Notes and Bonds

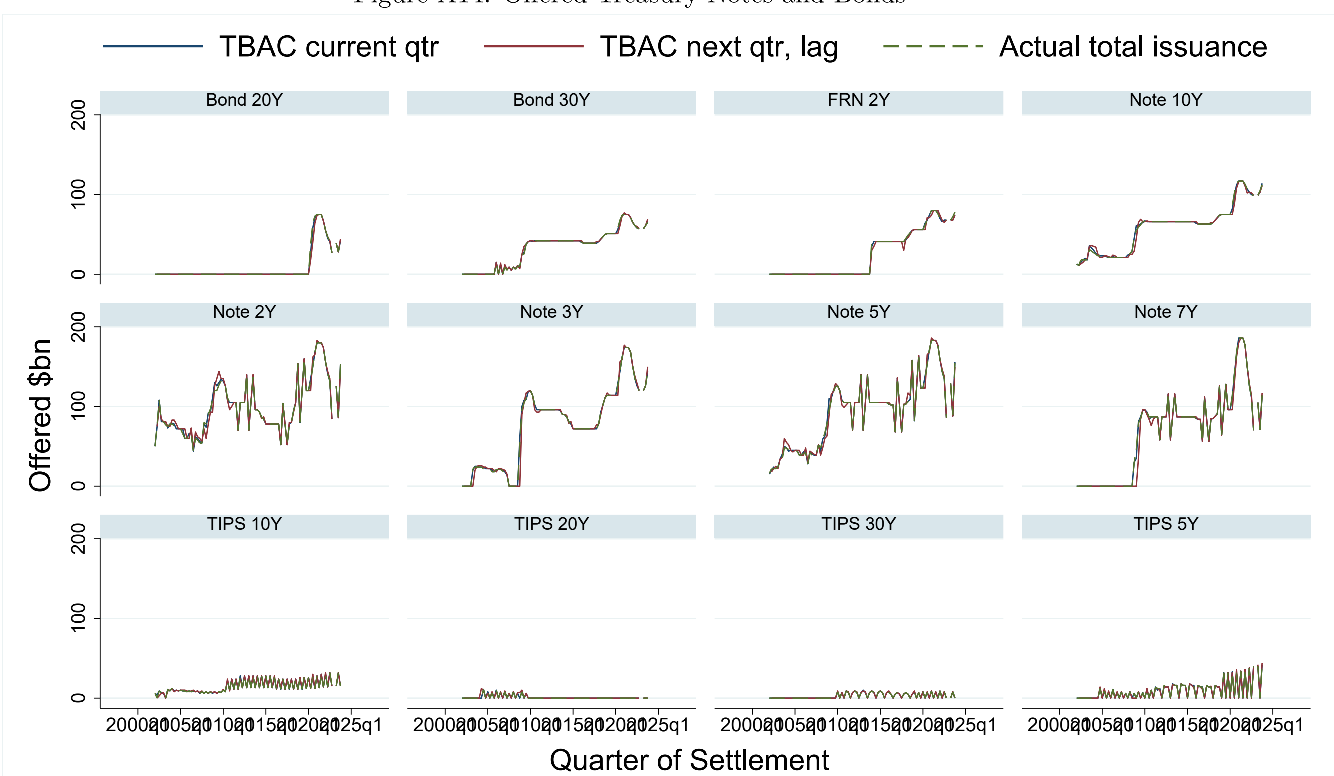


Figure A15: Offered 3-month Bills and 10-year Notes

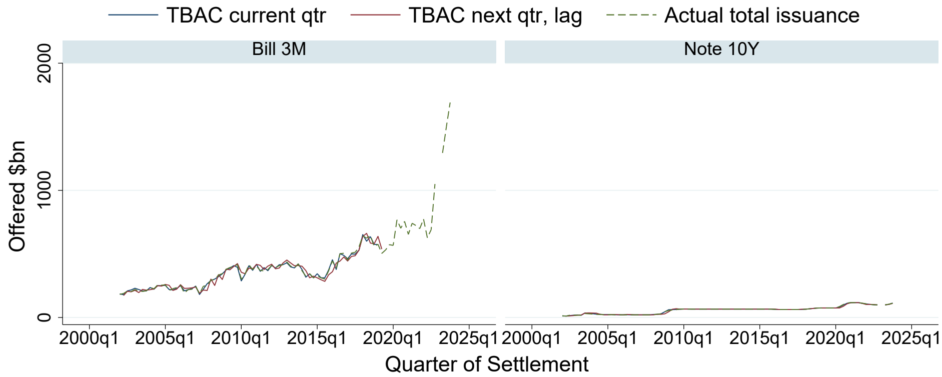


Figure A16: Surprise Distribution

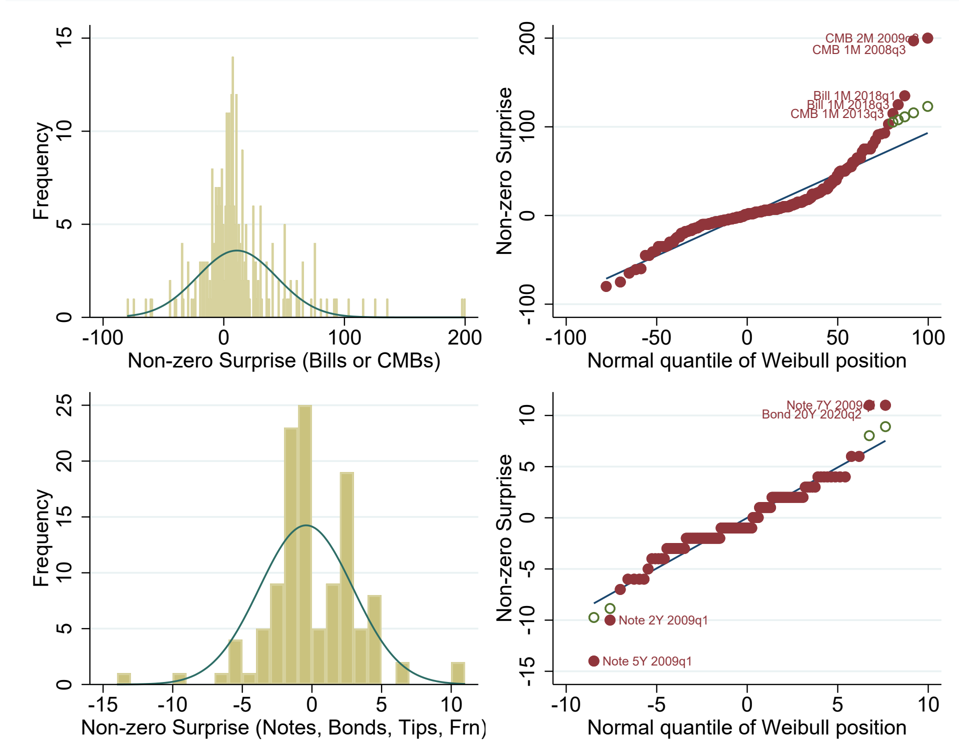


Figure A17: Update Distribution

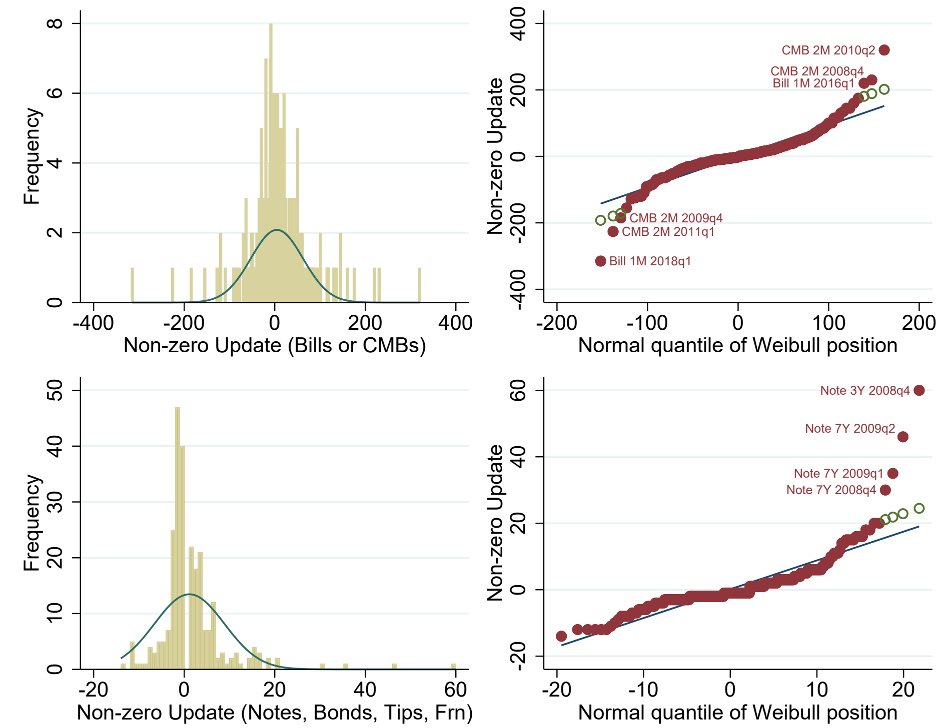


Figure A18: Updates and Surprises in 3-month Bill and 10-year Note Issuance

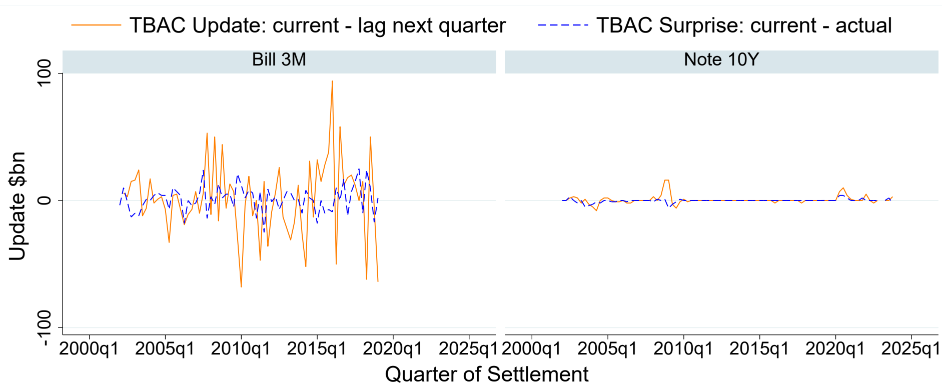


Figure A19: Updates and Surprises in Treasury Bill Issuance

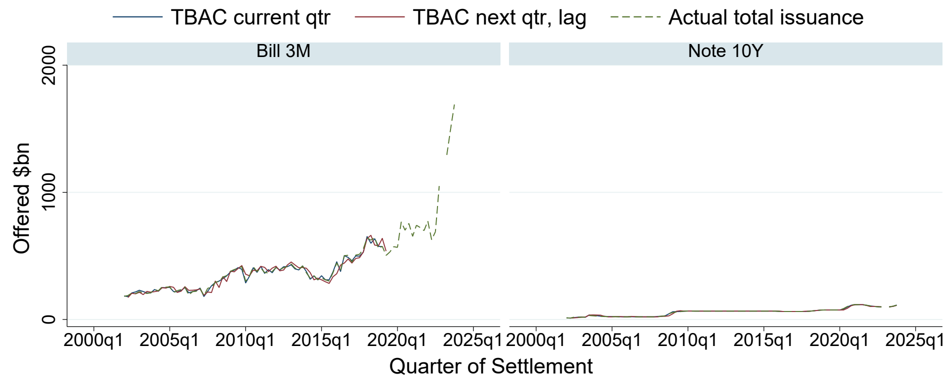
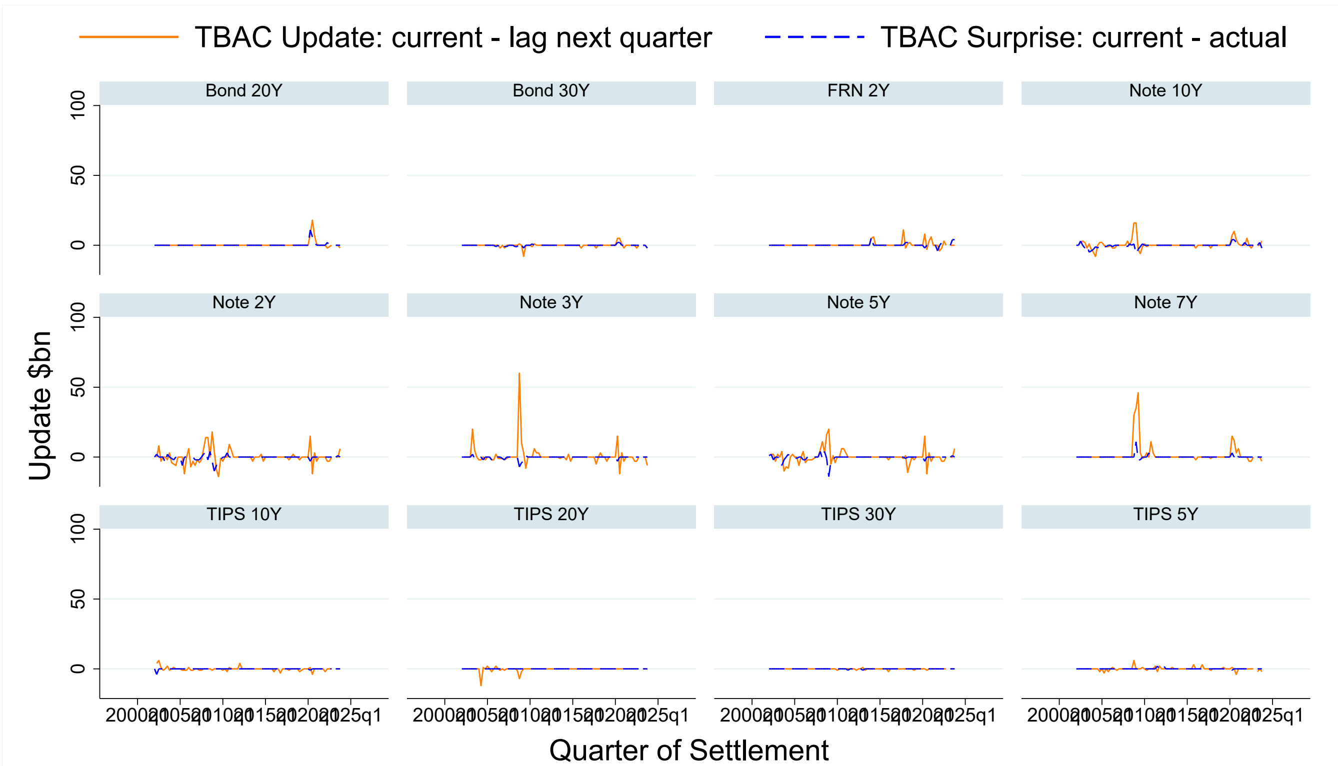


Figure A20: Updates and Surprises in Treasury Notes and Bond Issuance



B Appendix B: Overlapping Generations (OLG) Model of Government Debt Management Policy

Given that we do not observe extremely granular local supply effects in our empirical work, we present a simpler model than Vayanos and Vila (2021) without financial intermediaries.

An overlapping-generations (OLG) framework provides a natural microfoundation for the empirically observed sensitivity of term premia to debt-maturity composition because it captures imperfect risk-sharing across cohorts with finite horizons. Unlike moneyness or moral-hazard explanations, which rely on institutional frictions or strategic behavior, the OLG environment endogenously generates duration risk premia from heterogeneous saving horizons even under rational expectations.

To give theoretical intuition around the relationship between simple aggregate duration supply effects and risk premia. We derive this link from a limited investor horizon (in an OLG setup) and market incompleteness, in the form of lack of insurance for fluctuations in the resale value of long term bonds in turn driven by fluctuations in aggregate endowments.

B.1 Households

Households live for two periods and have CRRA preferences with risk aversion parameter γ and discount factor β. They receive an endowment $E_t \overset{iid}{\sim} F_E$ in period 1, which they allocate between consumption and savings in government bonds.

B.2 Government

The government issues two types of bonds and chooses the sequence of issuance across maturities, subject to a budget constraint that is ultimately backed by taxes levied on the younger generation.

1 Short-term bond: outstanding face value B_t^1, matures next period, at $t+1$.
Price per unit of face value at time t is P_t^1

2 Long-term bond: outstanding face value B_t^2, matures in 2 periods, at $t+2$.
Price per unit of face value at time t is P_t^2.
Price per unit of face value at time $t+1$ will be P_{t+1}^1, only distribution of it known at time t.

Definition of risk premium:

- Pure Expectation Hypothesis on simple returns[11]

$$P_t^2 = P_t^1 E_t[P_{t+1}^1]$$

- There is a risk premium when long-term bonds are priced at a discount below the expectation hypothesis implied price:

$$P_t^2 < P_t^1 E_t[P_{t+1}^1]$$

so defining $RP_t \equiv \frac{P_t^1 E_t[P_{t+1}^1]}{P_t^2} > 1$ and $rp_t = ln(RP_t)$ there is a risk premium when $RP_t > 1$, i.e. when $rp_t > 0$

[11] as in Campbell, Eqn. (8.18), rearranged.

In every period, the government issues N_t^1 new short bonds and N_t^2 new long bonds. The total outstanding face value supplied is $B_t^2 = N_t^2$ and $B_t^1 = B_{t-1}^2 + N_t^1$. Note that fixing any exogenous policy $\{N_t^1, N_t^2\}_{t\geq 0}$ the government also fixes $\{B_t^1, B_t^2\}_{t\geq 0}$, given initial $B_{-1}^2 = 0$. Taxes T_t adjust endogenously so that for any exogenous sequences $\{N_t^1, N_t^2\}_{t\geq 0}$, the budget below is balanced:

$$\overbrace{N_{t-2}^2 + N_{t-1}^1}^{\text{Present value of old debt maturing}} = \overbrace{P_t^1 N_t^1 + P_t^2 N_t^2}^{\text{Present value of new debt issued}} + \overbrace{T_t}^{\text{Taxes}}$$

Replacing $N_{t-2}^2 = B_{t-2}^2$ and $N_{t-1}^1 = B_{t-1}^1 - B_{t-2}^2$ and $N_t = B_t^1 - B_{t-1}^2$ and $N_t^2 = B_t^2$ we can express the constraint in terms of outstanding debt:

$$\underbrace{B_{t-1}^1 + P_t^1 B_{t-1}^2}_{\text{Present value of old outstanding debt}} = \underbrace{P_t^1 B_t^1 + P_t^2 B_t^2}_{\text{Present value of new outstanding debt}} + \underbrace{T_t}_{\text{Taxes}}$$

B.3 Key Results

The investor FOCs imply that in equilibrium:

$$rp_t = \gamma b_t \sigma_p^2$$

Hence the risk premium is increasing in:

- Risk Aversion γ
- Quantity risk σ_p (volatility of conjectured normal distribution for $ln(P_{t+1}^1) = -r_{t+1,t+2}$)
- Fraction of long term bonds: $b_t \equiv B_t^2/(B_t^1 + B_t^2)$

Assuming no change in σ_p^2, such a debt management operation that reduces b_t must reduce term risk premium.

One interesting fact is that central bank long-term asset purchases (quantitative easing) is functionally equivalent to a government debt management operation that reduces b_t. This is because if a central bank (such as the Federal Reserve) buys long-term bonds, it is reducing the supply of long-term bonds in the market and financing it by increasing the supply of bank reserves (which have low duration like short-term bonds). Hence, our model is functionally a model of quantitative easing as much as it is one of government debt management. Indeed, one of the stated goals of quantitative easing is reducing the term risk premium to stimulate more borrowing by reducing long-term interest rates in long-dated liabilities such as mortgages.

B.4 Household Portfolio Choice

$$\max_{B_t^1, B_t^2} \quad \frac{C_{t,young}^{1-\gamma}}{1-\gamma} + \beta E_t \left[\frac{C_{t+1,old}^{1-\gamma}}{1-\gamma} \right]$$

$$s.t.\ C_{t,young} = E_t - P_t^1 B_t^1 - P_t^2 B_t^2 - T_t$$

$$s.t.\ C_{t+1,old} = B_t^1 + P_{t+1}^1 B_t^2$$

We will suppress for brevity the "Young" and "Old" labels for the rest of this subsection. First order conditions are:

$$\partial_{B_t^1} = 0 \quad \Leftrightarrow \quad P_t^1 C_t^{-\gamma} = \beta E_t \left[C_{t+1}^{-\gamma} \right]$$
$$\partial_{B_t^2} = 0 \quad \Leftrightarrow \quad P_t^2 C_t^{-\gamma} = \beta E_t \left[C_{t+1}^{-\gamma} P_{t+1}^1 \right]$$

We postulate that for some appropriate F_E, the investor can rationally conjecture at time t that next resale price is log normal:

$$ln(P_{t+1}^1) \sim N(\mu_p, \sigma_p^2)$$

We verify this conjecture in Theorem 1 in the Appendix.

Lemma 1: Assuming $ln(P_{t+1}^1) = -r_{t+1,t+2}$ has distribution close to zero (and in particular $\mu_p^2, \sigma_p^4 \approx 0$), then C_{t+1} is approximatively log normal. Defining $b_t \equiv B_t^2/(B_t^2 + B_t^1) \in (0,1)$:

$$ln(C_{t+1}) \approx N(\overbrace{ln(B_t^1 + B_t^2) + \mu_p b_t + 0.5\sigma_p^2 b_t(1-b_t)}^{\equiv \mu_c}, \quad \overbrace{\sigma_p^2 b_t^2}^{\equiv \sigma_c^2})$$

Moreover, the approximation tends to exact as time intervals tend to zero (so is exact in continuous time).■

Then we can compute, using the properties of log normal distributions:

$$E_t \left[C_{t+1}^{-\gamma} \right] = e^{-\gamma \mu_c + 0.5\gamma^2 \sigma_c^2}$$

Lemma 2: After some computation:

$$E_t \left[C_{t+1}^{-\gamma} P_{t+1}^1 \right] = e^{-\gamma \mu_c + \mu_p + 0.5(1-2\gamma b_t)\sigma_p^2 + 0.5\gamma^2 \sigma_c^2} \quad ■$$

The taking the ratio of the second FOC to the first:

$$\frac{P_t^2}{P_t^1} = \frac{E_t \left[C_{t+1}^{-\gamma} P_{t+1}^1 \right]}{E_t \left[C_{t+1}^{-\gamma} \right]} = e^{\mu_p + 0.5(1-2\gamma b_t)\sigma_p^2}$$
$$= e^{\mu_p + 0.5\sigma_p^2} \quad e^{-\gamma b_t \sigma_p^2}$$
$$= E_t[P_{t+1}^1] \quad e^{-\gamma b_t \sigma_p^2}$$

Therefore the gross risk premium is:

$$RP_t = \frac{P_t^1 E_t[P_{t+1}^1]}{P_t^2} = e^{\gamma b_t \sigma_p^2}$$

Defining the risk premium rate rp_t as $e^{rp_t} = RP_t$, we get:

$$rp_t = \gamma b_t \sigma_p^2$$

So the risk premium is increasing in:

- Risk Aversion γ
- Quantity risk (volatility of resale price) σ_p
- Fraction of long term bonds: $b_t = B_t^2/(B_t^1 + B_t^2)$

A government debt management shock reducing the maturity of the term structure consists in the government some supply of long-term bonds into new supply of short-term bonds so the maturity is a reduction of b_t. In our model, this must reduce term risk premia.

B.5 Market Clearing and Unique Equilibrium

Since outstanding bond supply is exogenously fixed, equilibrium quantity is equal to supply. To find equilibrium prices, we fix B_t^1, B_t^2 into investor FOCs to be the supplied quantities and solve for equilibrium prices that make these quantities demanded. We have three equations (2 FOCs, Government Budget) for 3 endogenous variables (P_t^1, P_t^2, T_t) The first FOC is:

$$P_t^1 C_t^{-\gamma} = \beta e^{-\gamma\mu_c + 0.5\gamma^2\sigma_c^2}$$
$$P_t^1 [E_t - P_t^1 B_t^1 - P_t^2 B_t^2 - T_t]^{-\gamma} = \beta e^{-\gamma\mu_c + 0.5\gamma^2\sigma_c^2}$$

Replacing taxes with the government budget $T_t = B_{t-1}^1 + P_t^1 B_{t-1}^2 - (P_t^1 B_t^1 - P_t^2 B_t^2)$ we get:

$$P_t^1 [E_t - (B_{t-1}^1 + P_t^1 B_{t-1}^2)]^{-\gamma} = \beta e^{-\gamma\mu_c + 0.5\gamma^2\sigma_c^2}$$

This pins down a unique equilibrium $P_t^{1*} > 0$, assuming $E_t > B_{t-1}^1$.[12] Plugging this short term bond price in the second FOC (rewritten as ratio to the first), we recover the equilibrium price of long term bonds:

$$P_t^{2*} = P_t^{1*} e^{\mu_p + 0.5\sigma_p^2} \quad e^{-\gamma b_t \sigma_p^2}$$

And plugging prices back to the government budget we find equilibrium taxes:

$$T_t^* = B_{t-1}^1 + P_t^{1*} B_{t-1}^2 - (P_t^{1*} B_t^1 - P_t^{2*} B_t^2)$$

Finally, by Walras' law, we already know that the aggregate resource constraint holds: [13]

$$C_{t,young} + C_{t,old} = E_t$$

[12] the LHS is monotonically strictly increasing in P_t^1, going from $LHS = 0$ when $P_t^1 = 0$ to $LHS \to +\infty$ when $P_t^1 \to (E_t - B_{t-1}^1)/B_{t-2}^2$, while the RHS is a constant strictly above zero. So there must be one unique P_t^{1*} balancing the equation.

[13] If we want to check this directly, replace consumption with the budget constrain of the young generation old generation (the one born in $t-1$):

$$\overbrace{E_t - P_t^1 B_t^1 - P_t^2 B_t^2 - T_t}^{C_{t,young}} + \overbrace{B_{t-1}^1 + P_t^1 B_{t-1}^2}^{C_{t,old}} = E_t$$

Now replacing the government budget $T_t = -P_t^1 B_t^1 - P_t^2 B_t^2 + B_{t-1}^1 + P_t^1 B_{t-1}^2$, we get $E_t = E_t$, which clearly holds true.

B.6 Comparative Statics and Calibration

Fix any initial exogenous policy $\{B^1_\tau, B^2_\tau\}_{\tau\geq 0}$ and the corresponding equilibrium endogenous quantities $\{P^1_\tau, P^2_\tau, T_\tau, C_\tau\}_{\tau\geq 0}$, and a time t when a debt management operation is implemented.

Define a maturity shock as a temporary unexpected decrease in the share of outstanding long-term bonds from b^2_t to $\hat{b}^2_t \equiv q b^2_t$ (for some $q \in (0,1)$) To implement this, the new levels for $\hat{B}^1_t, \hat{B}^2_t$ are different from B^1_t, B^2_t, while for all other periods $\tau \neq t$ outstanding bonds remain unchanged:

$$
\begin{aligned}
\hat{B}^1_{t-1} &= B^1_{t-1}, & \hat{B}^2_{t-1} &= B^2_{t-1} \\
\hat{B}^1_{t} &\neq B^1_{t}, & \hat{B}^2_{t} &\neq B^2_{t} \\
\hat{B}^1_{t+1} &= B^1_{t+1}, & \hat{B}^2_{t+1} &= B^2_{t+1} \\
&\ldots & &\ldots
\end{aligned}
$$

Any such one-period deviation in outstanding bonds is easily achievable with two deviations in the issuance schedule. Table 8 shows the values of various parameters used in the calibration of the model.[14]

Table 8: Calibration of the OLG Model

Parameter	**Symbol**	**Value**
Risk aversion (CRRA)	γ	2.0
Discount factor	β	0.99
Mean endowment	$E[W_t]$	100
Std. dev. of endowment	$\text{Std}(W_t)$	5
Share of long bonds	b_t	0.50
Total bond supply	$B^1_t + B^2_t$	50.5
Shock multiplier	q	0.50
Growth mean (if used)	μ_η	1%
Growth volatility (if used)	σ_η	5%

B.6.1 Temporary maturity shift with no change total outstanding face value

This case is the easier to analyze, even though more unrealistic. Suppose that a government debt maturity shift is implemented such that total face value is unchanged:

$$
\frac{\hat{B}^2_t}{\hat{B}^1_t + \hat{B}^2_t} = q\frac{B^2_t}{B^1_t + B^2_t}
$$

$$
\hat{B}^1_t + \hat{B}^2_t = B^1_t + B^2_t
$$

[14] This can be achieved adjusting the issuance schedule at times t and (for short bond issuance only) $t+1$:

$$
\begin{aligned}
\hat{N}^1_{t-1} &= N^1_{t-1}, & \hat{N}^2_{t-1} &= N^2_{t-1} \\
\hat{N}^1_{t} &= N^1_{t} + (\hat{B}^1_t - B^1_t), & \hat{N}^2_{t} &= N^2_{t} + (\hat{B}^2_t - B^2_t) \\
\hat{N}^1_{t+1} &= N^1_{t+1} - (\hat{B}^2_t - B^2_t), & \hat{N}^2_{t+1} &= N^2_{t+1} \\
\hat{N}^1_{t+2} &= N^1_{t+2}, & \hat{N}^2_{t+2} &= N^2_{t+2} \\
&\ldots & &\ldots
\end{aligned}
$$

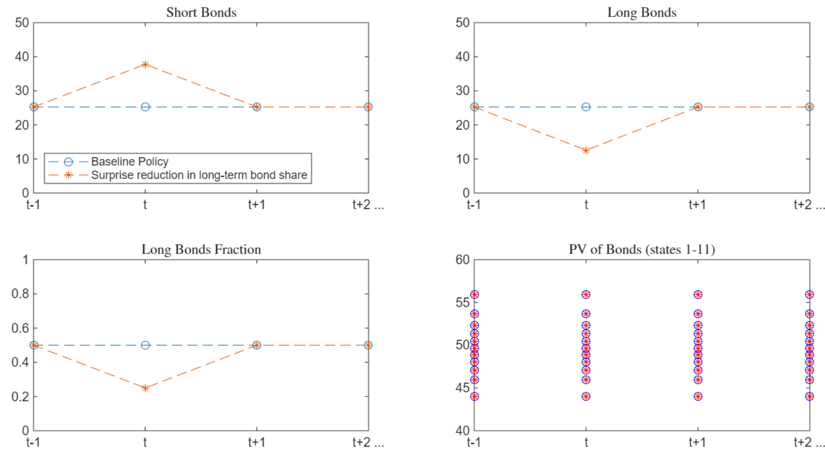
Applying the implicit function theorem to Theorem 1[15] , we can verify that the reduction in the share of long term debt from b_t to $\hat{b}_t = qb_t$ implies $\hat{\sigma}_b^2 > \sigma_b^2$, therefore we have an ambiguous effect on risk premia

$$rp_t = \gamma\sigma_p^2 b_t < \gamma\hat{\sigma}_p^2 b_t > \gamma\hat{\sigma}_p^2 \hat{b}_t = r\hat{p}_t$$

Figure A21 shows the evolution of short-term and long-term debt stocks and shares. Figure A22 decomposes the impact on long term rates. Figure A23 shows the impact on the government bond risk premium.

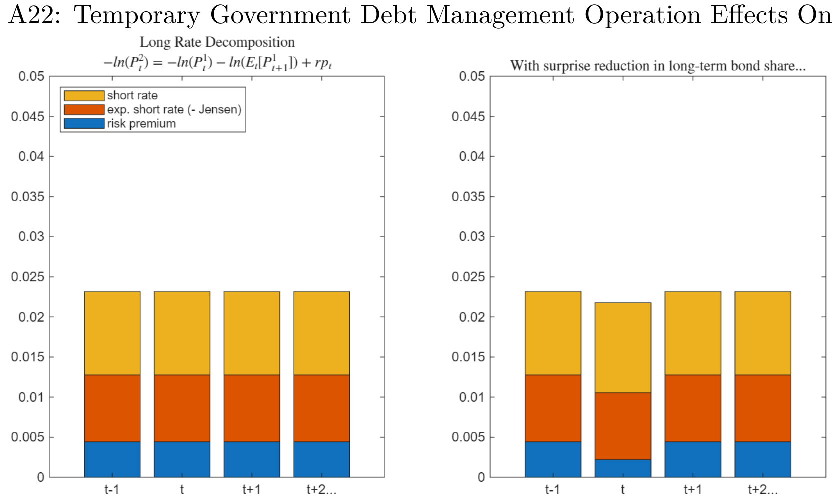
[15] One can rewrite the implicit solution for σ_p^2 of Theorem 1 in form $g(\sigma_p^2, b_t) = 0$, then compute $\frac{\partial \sigma_p^2}{\partial b_t} = -\partial_{b_t} g / \partial_{\sigma_p^2} g$. It is useful to notice that D remains unchanged, since bond quantities at and beyond $t + 1$ remain unchanged

Figure A21: Temporary Government Debt Management Operation Effects On Debt Issuance



Note: Simulated effects of a government debt-management operation that reduces the share of long-term bonds. Panels show resulting changes in short- and long-term debt stocks, yields, and risk premia under the calibrated OLG model. The model predicts that shortening maturities lowers term premia and borrowing costs.

Figure A22: Temporary Government Debt Management Operation Effects On Yields

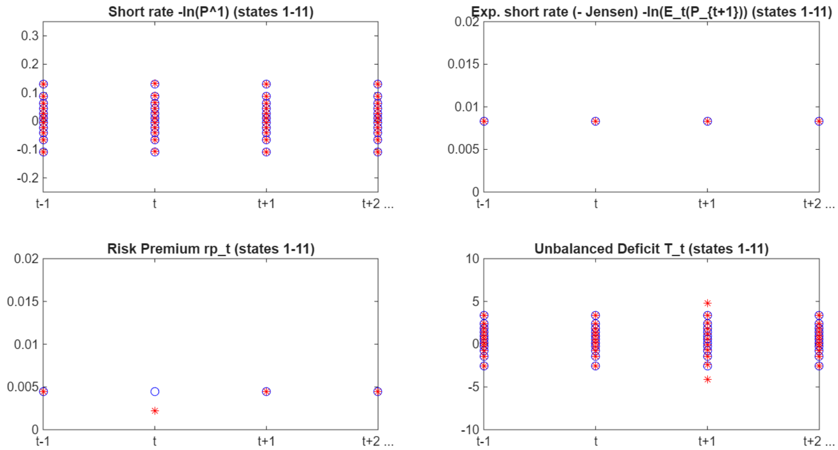


Note: Simulated effects of a government debt-management operation that reduces the share of long-term bonds. Panels show resulting changes in short- and long-term debt stocks, yields, and risk premia under the calibrated OLG model. The model predicts that shortening maturities lowers term premia and borrowing costs.

B.6.2 Permanent maturity shift with no change total outstanding face value

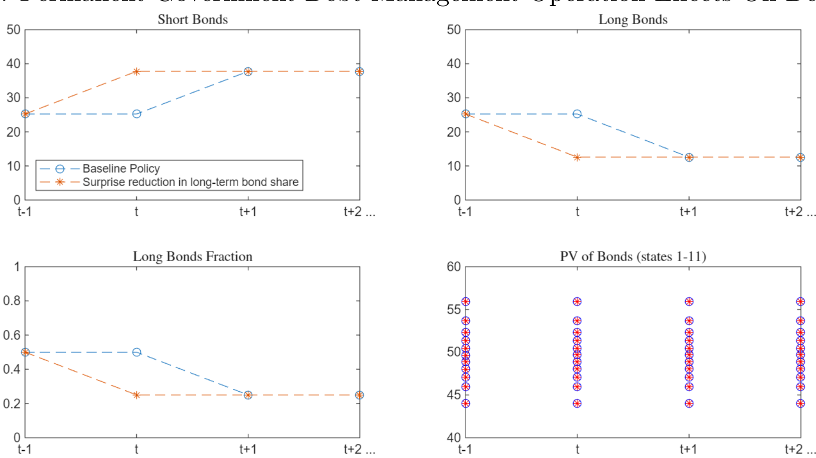
Now suppose the long bond share drops permanently to $\hat{b}_t^2 \equiv qb_t^2$ for some $q \in (0,1)$ and the total face value remains unchanged. Figure A24 shows the evolution of short-term and long-term debt stocks and shares. Figure A25 decomposes the impact on long term rates. Figure A26 shows the impact on the government bond risk premium.

Figure A23: Temporary Government Debt Management Operation Effects On Risk Premium



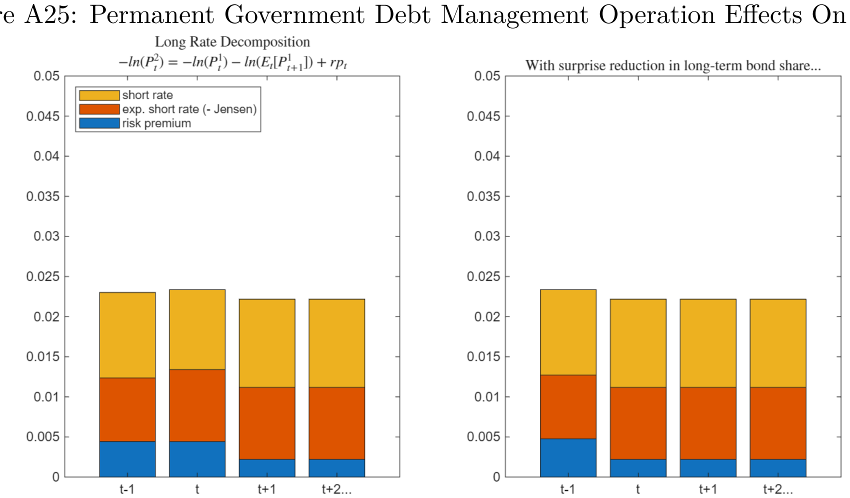
Note: Simulated effects of a government debt-management operation that reduces the share of long-term bonds. Panels show resulting changes in short- and long-term debt stocks, yields, and risk premia under the calibrated OLG model. The model predicts that shortening maturities lowers term premia and borrowing costs.

Figure A24: Permanent Government Debt Management Operation Effects On Debt Issuance



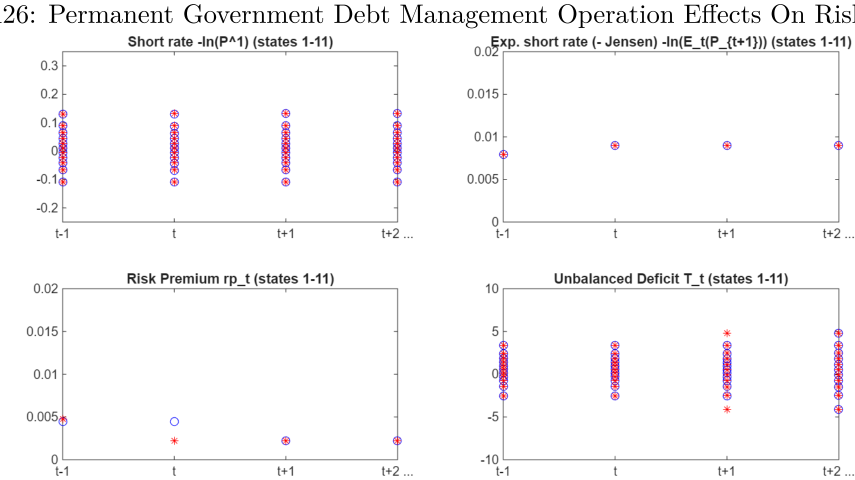
Note: Simulated effects of a government debt-management operation that reduces the share of long-term bonds. Panels show resulting changes in short- and long-term debt stocks, yields, and risk premia under the calibrated OLG model. The model predicts that shortening maturities lowers term premia and borrowing costs.

Figure A25: Permanent Government Debt Management Operation Effects On Yields



Note: Simulated effects of a government debt-management operation that reduces the share of long-term bonds. Panels show resulting changes in short- and long-term debt stocks, yields, and risk premia under the calibrated OLG model. The model predicts that shortening maturities lowers term premia and borrowing costs.

Figure A26: Permanent Government Debt Management Operation Effects On Risk Premium



Note: Simulated effects of a government debt-management operation that reduces the share of long-term bonds. Panels show resulting changes in short- and long-term debt stocks, yields, and risk premia under the calibrated OLG model. The model predicts that shortening maturities lowers term premia and borrowing costs.

B.6.3 Maturity shift with no change in present value of government debt

If current taxation stays unchanged $\hat{T}_t = T_t$, or, equivalently, the present value of outstanding government debt remains unchanged, then:

$$\frac{\hat{B}_t^2}{\hat{B}_t^1 + \hat{B}_t^2} = q\frac{B_t^2}{B_t^1 + B_t^2}$$
$$\hat{P}_t^1 \hat{B}_t^1 + \hat{P}_t^2 \hat{B}_t^2 = P_1^t B_t^1 + P_2^t B_t^2$$

We have already established that prices only depend on sum and ratios of outstanding face value. So we rewrite a debt management change in outstanding bonds in terms of changed sums and ratios of face values. The new ratio is by definition:

$$\hat{b}_t = qb_t$$

The new sum is pinned by the second equation, which after some algebra and using Lemma 2 can be rewritten as:

$$(\hat{B}_t^1 + \hat{B}_t^2) = (B_t^1 + B_t^2)\frac{P_1^t}{\hat{P}_t^1}\frac{(1-b_t) + b_t e^{\mu_p + 0.5\sigma_p^2} \quad e^{-\gamma b_t \sigma_p^2}}{(1-qb_t) + qb_t e^{\hat{\mu}_p + 0.5\hat{\sigma}_p^2} \quad e^{-\gamma q b_t \hat{\sigma}_p^2}}$$

This is an implicit function pinning $(\hat{B}_t^1 + \hat{B}_t^2)$, because $\hat{P}_t^1$, $\hat{\mu}_p$ and $\hat{\sigma}_p^2$ depend on $(\hat{B}_t^1 + \hat{B}_t^2)$ (and on $\hat{b}_t = qb_t$, as well as the on remaining unchanged schedule) in the way already shown in the previous two subsections 1.6 and 1.7.

Note that even if we were able to sign the relationship[16] between $(\hat{B}_t^1 + \hat{B}_t^2)$ and $(B_t^1 + B_t^2)$, the effect on risk premia would remain ambiguous due to the positive effect on σ_p^2 of a decrease from b_t to qb_t.

C Appendix C: Model Extensions

C.1 Model Stationary in Growth Rate

Suppose instead that endowment growth is log-normal $\eta_{t+1} \equiv ln(E_{t+1}/E_t) \sim N(\mu_\eta, \sigma_\eta^2)$. For intuition, notice that this is analogous to assuming a normal net growth rate $(E_{t+1} - E_t)/E_t$, if this has small absolute value, i.e if μ_η, σ_η^2 are close to zero.[17] Then, at time t, the belief on next endowment is:

$$\begin{aligned} ln(E_{t+1}) &\sim ln(E_t) + N(\mu_\eta, \sigma_\eta^2) \\ &\sim N(ln(E_t) + \mu_\eta, \sigma_\eta^2) \end{aligned}$$

So defining $\mu_{E,t} = ln(E_t) + \mu_\eta$ and $\sigma_E = \sigma_\eta^2$, we have $ln(E_{t+1}) \sim N(\mu_{E,t}, \sigma_E^2)$. Notice results up to Lemma 2 are unchanged. In Theorem 1, we just replace μ_E with $\mu_{E,t}$. Comparative statics on risk premia are unchanged. Still, to find formulas to solve for steady state on a computer, we want to solve a normalized model.

[16] our intuition is that since a larger share of face value must be in short-term bonds (which have higher price than long term bonds), while total PV remains unchanged, total face value must decrease

[17] $ln(E_{t+1}/E_t) = ln(1 + (E_{t+1} - E_t)/E_t) \approx (E_{t+1} - E_t)/E_t \sim N(\mu_\eta, \sigma_\eta^2)$

C.2 Model Normalized by E_{t-1}

Define normalized variables as variables divided by old endowment:

$$\bar{E}_t = E_t/E_{t-1}$$
$$\bar{C}_{t,young} = C_{t,young}/E_{t-1}$$
$$\bar{C}_{t,old} = C_{t,old}/E_{t-1}$$
$$\bar{B}^1_t = B^1_t/E_{t-1}$$
$$\bar{B}^2_t = B^2_t/E_{t-1}$$
$$\bar{T}_t = T_t/E_{t-1}$$

We now assume that the government chooses a deterministic path for $\{\bar{B}^1_t, \bar{B}^2_t\}, :$ this is a change in the definition of issuance policy. The government budget becomes:

$$(\bar{E}_{t-1})^{-1}(\bar{B}^1_{t-1} + P^1_t \bar{B}^2_{t-1}) = P^1_t \bar{B}^1_t + P^2_t \bar{B}^2_t + \bar{T}_t$$

The investor problem becomes:

$$\max_{B^1_t, B^2_t} \quad \frac{\bar{C}^{1-\gamma}_{t,young}}{1-\gamma} + \beta E_t \left[\frac{(\bar{C}_{t+1,old}\bar{E}_t)^{1-\gamma}}{1-\gamma} \right]$$
$$s.t.\ \bar{C}_{t,young} = \bar{E}_t - P^1_t \bar{B}^1_t - P^2_t \bar{B}^2_t - \bar{T}_t$$
$$s.t.\ C_{t+1,old}\bar{E}_t = \bar{B}^1_t + P^1_{t+1}\bar{B}^2_t$$

The FOCs become:

$$\partial_{B^1_t} = 0 \quad \Leftrightarrow \quad P^1_t \bar{C}^{-\gamma}_t = \beta E_t \left[(\bar{C}_{t+1}\bar{E}_t)^{-\gamma} \right]$$
$$\partial_{B^2_t} = 0 \quad \Leftrightarrow \quad P^2_t \bar{C}^{-\gamma}_t = \beta E_t \left[(\bar{C}_{t+1}\bar{E}_t)^{-\gamma} P^1_{t+1} \right]$$

Postulating that the investor can rationally conjecture at time t that next resale price is log normal:

$$ln(P^1_{t+1}) \sim N(\mu_p, \sigma^2_p)$$

Then Lemma 1 becomes:

Lemma 1: Assuming $ln(P^1_{t+1}) = -r_{t+1,t+2}$ has distribution close to zero (and in particular $\mu^2_p, \sigma^4_p \approx 0$), then C_{t+1} is approximatively log normal. Defining $\bar{b}_t = b_t \equiv B^2_t/(B^2_t + B^1_t) \in (0,1)$:

$$ln(\bar{C}_{t+1}\bar{E}_t) \approx N(ln(\bar{B}^1_t + \bar{B}^2_t) + \mu_p b_t + 0.5\sigma^2_p b_t(1-b_t), \qquad \sigma^2_p b^2_t)$$
$$ln(\bar{C}_{t+1}) \approx N(\overbrace{ln(\bar{B}^1_t + \bar{B}^2_t) + \mu_p b_t + 0.5\sigma^2_p b_t(1-b_t) - ln(\bar{E}_t)}^{\equiv \mu_{\bar{c}}}, \qquad \overbrace{\sigma^2_p b^2_t}^{\equiv \sigma^2_{\bar{c}}})$$

Notice that we can easily verify this is equivalent to our pre-existing Lemma 1 because $ln(\bar{C}_{t+1}) = ln(C_{t+1}) - ln(E_t)$ and $ln(\bar{B}^1_t + \bar{B}^2_t) - ln(\bar{E}_t) = ln(B^1_t + B^2_t) - ln(E_t)$, so $\mu_{\bar{c}} = \mu_c - ln(E_t)$ and $\sigma^2_{\bar{c}} = \sigma^2_c$ Then we can compute, using the properties of log normal distributions:

$$E_t\left[(\bar{C}_{t+1}\bar{E}_t)^{-\gamma}\right] = e^{-\gamma(\mu_{\bar{c}}+ln(\bar{E}_t))+0.5\gamma^2\sigma_c^2}$$

Where $\mu_{\bar{c}} + ln(\bar{E}_t) = ln(\bar{B}_t^1 + \bar{B}_t^2) + \mu_p b_t + 0.5\sigma_p^2 b_t(1-b_t) \equiv \mu_{\bar{c\eta}}$

Lemma 2: After some computation:

$$E_t\left[(\bar{C}_{t+1}\bar{E}_t)^{-\gamma}P_{t+1}^1\right] = e^{-\gamma(\mu_{\bar{c}}+ln(\bar{E}_t))+0.5\gamma^2\sigma_c^2+\mu_p+0.5\sigma_p^2-\gamma b_t\sigma_p^2}$$

This also can be simply obtained dividing both sides of Lemma 2. The ratio of the FOCs is again:

$$\frac{P_t^2}{P_t^1} = \frac{E_t\left[(\bar{C}_{t+1}\bar{E}_t)^{-\gamma}P_{t+1}^1\right]}{E_t\left[(\bar{C}_{t+1}\bar{E}_t)^{-\gamma}\right]} = e^{\mu_p+0.5\sigma_p^2} \quad e^{-\gamma b_t\sigma_p^2} = E_t[P_{t+1}^1] \quad e^{-\gamma b_t\sigma_p^2}$$

Therefore as expected we have again

$$rp_t = \gamma b_t \sigma_p^2$$

Equilibrium is given by prices making the FOCs hold at given supply (and by gov budget):

$$\begin{aligned}
&P_t^1\left[\bar{E}_t - (\bar{E}_{t-1})^{-1}(\bar{B}_{t-1}^1 + P_t^1\bar{B}_{t-1}^2)\right]^{-\gamma} = \beta e^{-\gamma\mu_{\bar{c\eta}}+0.5\gamma^2\sigma_c^2}\\
&P_t^2 = P_t^1 e^{\mu_p+0.5\sigma_p^2-\gamma b_t\sigma_p^2}\\
&\bar{T}_t = (\bar{E}_{t-1})^{-1}(\bar{B}_{t-1}^1 + P_t^1\bar{B}_{t-1}^2) - P_t^1\bar{B}_t^1 - P_t^2\bar{B}_t^2
\end{aligned}$$

Notice $\mu_{\bar{c\eta}} = \mu_{\bar{c}} + ln(\bar{E}_t) = \mu_c - ln(E_t) + ln(\bar{E}_t) = \mu_c - ln(E_{t-1})$, so indeed the first FOC is simply the original FOC divided on both sides by $E_{t-1}^{-\gamma}$

C.2.1 Normalized Theorem 1

Define

$$\begin{aligned}
\bar{D}^\gamma &\equiv \beta e^{-\gamma\tilde{\mu}_{\bar{c\eta}}+0.5\gamma^2\tilde{\sigma}_{\bar{c\eta}}^2}\\
&= \beta e^{-\gamma\left[ln(\bar{B}_{t+1}^1+\bar{B}_{t+1}^2)+\tilde{\mu}_p b_{t+1}+0.5\tilde{\sigma}_p^2 b_{t+1}(1-b_{t+1})\right]+0.5\gamma^2\tilde{\sigma}_p^2 b_{t+1}^2}\\
&= \beta E_t^\gamma e^{-\gamma\left[ln(B_{t+1}^1+B_{t+1}^2)+\tilde{\mu}_p b_{t+1}+0.5\tilde{\sigma}_p^2 b_{t+1}(1-b_{t+1})\right]+0.5\gamma^2\tilde{\sigma}_p^2 b_{t+1}^2}\\
&= E_t^\gamma D^\gamma
\end{aligned}$$

So $\bar{D} = E_t D$. Also define:

$$\bar{A} = (\bar{B}_t^1 + \bar{B}_t^2)e^{0.5\sigma_p^2 b_t(1-b_t)} = E_{t-1}^{-1}A$$

Thus $AD = \bar{A}\bar{D}\bar{E}_t^{-1}$. Replacing in the result of Theorem 1, along with $ln(D) = ln(\bar{D}) - ln(E_t)$

$$ln(P^1_{t+1}) \approx \frac{(\bar{A}\bar{D}\bar{E}_t^{-1}+1)}{\left[\bar{A}\bar{D}\bar{E}_t^{-1}b_t+\gamma^{-1}\right]}\left[-\frac{\bar{A}\bar{D}\bar{E}_t^{-1}}{2(\bar{A}\bar{D}\bar{E}_t^{-1}+1)^2}(b_t-\gamma^{-1})^2\sigma_p^2 + \underbrace{ln(\bar{E}_{t+1})}_{ln(E_{t+1})-ln(E_t)} + ln(\bar{D}) - ln(\bar{A}\bar{D}\bar{E}_t^{-1}+1)\right]$$

Since, at time t, $ln(barE_{t+1}) \sim N(\mu_\eta, \sigma_\eta^2)$ and the rest is deterministic, then $ln(P_{t+1}) \overset{approx}{\sim} N(\mu_p, \sigma_p^2)$ with

$$\mu_p = \frac{(\bar{A}\bar{D}\bar{E}_t^{-1}+1)}{\left[\bar{A}\bar{D}\bar{E}_t^{-1}b_t+\gamma^{-1}\right]}\left[-\frac{\bar{A}\bar{D}\bar{E}_t^{-1}(b_t-\gamma^{-1})^2}{2\left[\bar{A}\bar{D}\bar{E}_t^{-1}b_t+\gamma^{-1}\right]^2}\sigma_\eta^2 + \mu_\eta + ln(\bar{D}) - ln(\bar{A}\bar{D}\bar{E}_t^{-1}+1)\right]$$
$$\sigma_p^2 = \sigma_\eta^2\frac{(\bar{A}\bar{D}\bar{E}_t^{-1}+1)^2}{\left[\bar{A}\bar{D}\bar{E}_t^{-1}b_t+\gamma^{-1}\right]^2}$$

C.3 Model Normalized by E_t

Define the log-normal endowment growth $\Gamma_{t+1} \equiv E_{t+1}/E_t = e^{\eta_{t+1}}$ and define normalized variables as variables divided by the current endowment:

$$\bar{C}_{t,young} = C_{t,young}/E_t$$
$$\bar{C}_{t,old} = C_{t,old}/E_t$$
$$\bar{B}_t^1 = B_t^1/E_t$$
$$\bar{B}_t^2 = B_t^2/E_t$$
$$\bar{T}_t = T_t/E_t$$

Note: policy now sets $\{\bar{B}_t^1, \bar{B}_t^2\}$: we are now assuming that bond to current endowment levels are deterministic. This is a substantial change in the definition of issuance policy. Now let us rewrite all previously derived equations in terms of these new variables. The government budget becomes, dividing on both sides by E_t:

$$(\Gamma_t)^{-1}(\bar{B}_{t-1}^1 + P_t^1 \bar{B}_{t-1}^2) = P_t^1 \bar{B}_t^1 + P_t^2 \bar{B}_t^2 + \bar{T}_t$$

The FOCs can be rewritten, dividing on both sides:

$$\partial_{B_t^1} = 0 \quad \Leftrightarrow \quad P_t^1 \bar{C}_t^{-\gamma} = \beta E_t\left[(\bar{C}_{t+1}\Gamma_{t+1})^{-\gamma}\right]$$
$$\partial_{B_t^2} = 0 \quad \Leftrightarrow \quad P_t^2 \bar{C}_t^{-\gamma} = \beta E_t\left[(\bar{C}_{t+1}\Gamma_{t+1})^{-\gamma} P_{t+1}^1\right]$$

Notice that these can also be re-derived from the normalized investor problem, which we re-write below to show the normalized budget constraints:

$$\max_{B_t^1, B_t^2} \quad \frac{\bar{C}_{t,young}^{1-\gamma}}{1-\gamma} + \beta E_t\left[\frac{(\bar{C}_{t+1,old}\Gamma_{t+1})^{1-\gamma}}{1-\gamma}\right]$$
$$s.t.\ \bar{C}_{t,young} = 1 - P_t^1 \bar{B}_t^1 - P_t^2 \bar{B}_t^2 - \bar{T}_t$$
$$s.t.\ \bar{C}_{t+1,old}\Gamma_{t+1} = \bar{B}_t^1 + P_{t+1}^1 \bar{B}_t^2$$

Lemma 1 was, keeping the conjecture $ln(P_{t+1}^1) \sim N(\mu_p, \sigma_p^2)$:

$$ln\left(C_{t+1}\right) \approx ln\left(B_t^2 + B_t^1\right) + ln(P_{t+1}^1) b_t + 0.5\sigma_p^2 b_t(1-b_t)$$

So replacing the definitions of normalized variables and noting $\bar{b}_t = b_t$ we can rewrite it as:

$$ln\left(\bar{C}_{t+1}\Gamma_{t+1}\right) \approx ln\left(\bar{B}_t^2 + \bar{B}_t^1\right) + ln(P_{t+1}^1) b_t + 0.5\sigma_p^2 b_t(1-b_t)$$

Notice that this can also simply be re-derived following the same steps of the riginal Lemma 1 on the normalized old budget. So we have $ln(\bar{C}_{t+1}\Gamma_{t+1}) \sim N(\overbrace{ln(\bar{B}_t^1 + \bar{B}_t^2) + \mu_p b_t + 0.5\sigma_p^2 b_t(1-b_t)}^{\equiv \mu_{(\bar{c}\eta)}}, \overbrace{\sigma_p^2 b_t^2}^{\equiv \sigma^2_{(\bar{c}\eta)}})$ It is easy to show $\mu_{(\bar{c}\eta)} = \mu_{\bar{c}} + \mu_\eta$

and $\sigma^2_{(\bar{c}\eta)} = \sigma^2_{\bar{c}} - \sigma^2_\eta + 2b_t cov(\eta_{t+1}, ln(P_{t+1}))$

$$E_t\left[(\bar{C}_{t+1}\bar{\Gamma}_{t+1})^{-\gamma}\right] = e^{-\gamma\mu_{(\bar{c}\eta)}+0.5\gamma^2\sigma^2_{(\bar{c}\eta)}}$$

Lemma 2 becomes

$$E_t\left[(\bar{C}_{t+1}\Gamma_{t+1})^{-\gamma}P^1_{t+1}\right] = e^{-\gamma\mu_{(\bar{c}\eta)}+0.5\gamma^2\sigma^2_{(\bar{c}\eta)}+\mu_p+0.5\sigma^2_p-\gamma b_t\sigma^2_p}$$

So as expected we still have the ratio of FOCs giving the same result on risk premium:

$$\frac{P^2_t}{P^1_t} = \frac{E_t\left[(\bar{C}_{t+1}\Gamma_{t+1})^{-\gamma}P^1_{t+1}\right]}{E_t\left[(\bar{C}_{t+1}\Gamma_{t+1})^{-\gamma}\right]} = e^{\mu_p+0.5\sigma^2_p} \quad e^{-\gamma b_t\sigma^2_p} = E_t[P^1_{t+1}] \quad e^{-\gamma b_t\sigma^2_p}$$

$$rp_t = \gamma b_t\sigma^2_p$$

And the equilibrium prices and taxes are given by the normalized FOCs and normalized government budget. FOCs:

$$P^1_t\left[1-(\Gamma_t)^{-1}(\bar{B}^1_{t-1}+P^1_t\bar{B}^2_{t-1})\right]^{-\gamma} = \beta e^{-\gamma\mu_{(\bar{c}\eta)}+0.5\gamma^2\sigma^2_{(\bar{c}\eta)}} = \beta e^{-\gamma\left[ln(\bar{B}^1_t+\bar{B}^2_t)+\mu_p b_t+0.5\sigma^2_p b_t(1-b_t)\right]+0.5\gamma^2\sigma^2_p b^2_t}$$
$$P^2_t = P^1_t e^{\mu_p+0.5\sigma^2_p} \quad e^{-\gamma b_t\sigma^2_p}$$
$$\bar{T}_t = (\Gamma_t)^{-1}(\bar{B}^1_{t-1}+P^1_t\bar{B}^2_{t-1}) - P^1_t\bar{B}^1_t - P^2_t\bar{B}^2_t$$

It is easy to see the second equilibrium condition is unchanged, the first could be simply derived dividing both sides of the original equilibrium condition by $E_t^{-\gamma}$, and the third dividing both sides of the gov budget by E_t.

C.3.1 Normalized Theorem 1

Define

$$\begin{aligned}\bar{D}^\gamma &\equiv \beta e^{-\gamma\tilde{\mu}_{\bar{c}\eta}+0.5\gamma^2\tilde{\sigma}^2_{\bar{c}\eta}}\\ &= \beta e^{-\gamma\left[ln(\bar{B}^1_{t+1}+\bar{B}^2_{t+1})+\tilde{\mu}_p b_{t+1}+0.5\tilde{\sigma}^2_p b_{t+1}(1-b_{t+1})\right]+0.5\gamma^2\tilde{\sigma}^2_p b^2_{t+1}}\\ &= \beta E^\gamma_{t+1} e^{-\gamma\left[ln(B^1_{t+1}+B^2_{t+1})+\tilde{\mu}_p b_{t+1}+0.5\tilde{\sigma}^2_p b_{t+1}(1-b_{t+1})\right]+0.5\gamma^2\tilde{\sigma}^2_p b^2_{t+1}}\\ &= E^\gamma_{t+1}D^\gamma\end{aligned}$$

So $\bar{D} = E_{t+1}D$. Also define:

$$\bar{A} = (\bar{B}^1_t+\bar{B}^2_t)e^{0.5\sigma^2_p b_t(1-b_t)} = E_t^{-1}A$$

Thus $AD = \bar{A}\bar{D}\Gamma^{-1}_{t+1}$. Also note: $ln(D) + ln(E_{t+1}) = ln(\bar{D})$ In Theorem 1 we reached, before approximating the transfiormation of log-normal:

$ln\left(1+DAe^{ln(P^1_{t+1})(b_t-\gamma^{-1})}\right) \approx ln(E_{t+1}) + ln(D) - \gamma^{-1}ln(P^1_{t+1})$. Replacing the new deterministic variables:

$$ln\left(1+\bar{A}\bar{D}\Gamma^{-1}_{t+1}e^{ln(P^1_{t+1})(b_t-\gamma^{-1})}\right) \approx ln(\bar{D}) - \gamma^{-1}ln(P^1_{t+1})$$
$$ln\left(1+\bar{A}\bar{D}e^{-ln(\Gamma_{t+1})+ln(P^1_{t+1})(b_t-\gamma^{-1})}\right) \approx ln(\bar{D}) - \gamma^{-1}ln(P^1_{t+1})$$

Let us define $Cov\left(ln(\Gamma_{t+1}), ln(P^1_{t+1})\right) \equiv \sigma_{\eta,p}$ and define $x \equiv -ln(\bar{\Gamma}_{t+1}) + ln(P^1_{t+1})(b_t-\gamma^{-1})$ with

$$E[x] = -\mu_\eta + \mu_p(b_t-\gamma^{-1})$$
$$V[x] = \sigma^2_\eta + \sigma^2_p(b_t-\gamma^{-1})^2 + 2\sigma_{\eta,p}(b_t-\gamma^{-1})$$

We assume $V[x^2]$ and $E[x]^2$ are small (this is a modified version of assumption A3), we can approximate with $x^2 \approx V[x]$ for values of x close to $x=0$ and replace this in the usual 2nd order Taylor approximation around $x=0$ of $f(x) = ln(1+\alpha e^x) \approx ln(\alpha+1) + \frac{\alpha}{\alpha+1}x + \frac{\alpha}{2(\alpha+1)^2}x^2 + o(x^2)$ with $\alpha = \bar{A}\bar{D}$. Replacing on the LHS:

$$ln(\bar{A}\bar{D}+1) + \frac{\bar{A}\bar{D}}{\bar{A}\bar{D}+1}\left[-ln(\Gamma_{t+1}) + ln(P^1_{t+1})(b_t-\gamma^{-1})\right] + \frac{\bar{A}\bar{D}}{2(\bar{A}\bar{D}+1)^2}\left[\sigma^2_\eta + \sigma^2_p(b_t-\gamma^{-1})^2 + 2\sigma_{\eta,p}(b_t-\gamma^{-1})\right]$$
$$\approx ln(\bar{D}) - \gamma^{-1}ln(P^1_{t+1})$$

Solving this linear equation for $ln(P^1_{t+1})$:

$$ln(P^1_{t+1}) \approx$$
$$\frac{\bar{A}\bar{D}+1}{\bar{A}\bar{D}b_t+\gamma^{-1}}\left[\frac{-\bar{A}\bar{D}}{2(\bar{A}\bar{D}+1)^2}\left[\sigma^2_\eta + \sigma^2_p(b_t-\gamma^{-1})^2 + 2\sigma_{\eta,p}(b_t-\gamma^{-1})\right] + \frac{\bar{A}\bar{D}}{\bar{A}\bar{D}+1}ln(\Gamma_{t+1}) + ln(\bar{D}) - ln(\bar{A}\bar{D}+1)\right]$$

Since on the RHS is a linear function of deterministic variables and $ln(\Gamma_{t+1})$, which is normal, then $ln(P^1_{t+1})$ must have (approx.) normal distribution, verifying our conjecture:

$$\Rightarrow ln(P^1_{t+1}) \overset{approx}{\sim} N(\mu_p, \sigma^2_p)$$

We can solve for its moments more explicitly. Start finding:

$$\sigma_{\eta,p} = Cov\left(ln(\Gamma_{t+1}), ln(P^1_{t+1})\right) = Cov\left(ln(\Gamma_{t+1}), \frac{\bar{A}\bar{D}}{\bar{A}\bar{D}b_t+\gamma^{-1}}ln(\Gamma_{t+1})\right) = \frac{\bar{A}\bar{D}}{\bar{A}\bar{D}b_t+\gamma^{-1}}\sigma^2_\eta$$

And:

$$\sigma^2_p = V\left(ln(P^1_{t+1})\right) = \left(\frac{\bar{A}\bar{D}}{\bar{A}\bar{D}b_t+\gamma^{-1}}\right)^2\sigma^2_\eta$$

Notice how σ^2_p is implicitly pinned[18] by the equation above, since it also appears in $\bar{A}$. Replacing $\sigma_{\eta,p}$ and σ^2_p out:

$$ln(P^1_{t+1}) \approx$$
$$\frac{\bar{A}\bar{D}+1}{\bar{A}\bar{D}b_t+\gamma^{-1}}\left[\frac{-\bar{A}\bar{D}}{2(\bar{A}\bar{D}+1)^2}\left[\sigma^2_\eta + \frac{(\bar{A}\bar{D})^2(b_t-\gamma^{-1})^2}{(\bar{A}\bar{D}b_t+\gamma^{-1})^2}\sigma^2_\eta + \frac{2\bar{A}\bar{D}(b_t-\gamma^{-1})}{\bar{A}\bar{D}b_t+\gamma^{-1}}\sigma^2_\eta\right] + \frac{\bar{A}\bar{D}}{\bar{A}\bar{D}+1}ln(\Gamma_{t+1}) + ln(\bar{D}) - ln(\bar{A}\bar{D}+1)\right]$$

[18] The solution exists unique as $LHS = 0 < RHS$ for $\sigma_p = 0$, while $LHS \to \infty > \sigma^2_\eta \leftarrow RHS$ for $\sigma_p \to \infty$)

Collecting σ_η^2

$$ln(P_{t+1}^1) \approx$$
$$\frac{\bar{A}\bar{D}+1}{\bar{A}\bar{D}b_t+\gamma^{-1}}\left[\frac{-\bar{A}\bar{D}}{2(\bar{A}\bar{D}+1)^2}\left[\frac{4\bar{A}\bar{D}b_t(b_t+\gamma^{-1})+\gamma^{-2}(\bar{A}\bar{D}-1)^2}{(\bar{A}\bar{D}b_t+\gamma^{-1})^2}\right]\sigma_\eta^2+\frac{\bar{A}\bar{D}}{\bar{A}\bar{D}+1}ln(\Gamma_{t+1})+ln(\bar{D})-ln(\bar{A}\bar{D}+1)\right]$$

Hence:

$$\mu_p = E[ln(P_{t+1}^1)] = -\bar{A}\bar{D}\left[\frac{4\bar{A}\bar{D}b_t(b_t+\gamma^{-1})+\gamma^{-2}(\bar{A}\bar{D}-1)^2}{2(\bar{A}\bar{D}+1)(\bar{A}\bar{D}b_t+\gamma^{-1})^3}\right]\sigma_\eta^2+\frac{\bar{A}\bar{D}}{(\bar{A}\bar{D}b_t+\gamma^{-1})}\mu_\eta+\frac{\bar{A}\bar{D}+1}{\bar{A}\bar{D}b_t+\gamma^{-1}}\left[ln(\bar{D})-ln(\bar{A}\bar{D}+1)\right]$$

D Appendix D: Proofs

D.1 Proof of Lemma 1:

From the old-age budget:

$$\frac{C_{t+1,old}}{B_t^1} = 1+\frac{B_t^2}{B_t^1}P_{t+1}^1$$
$$ln\left(\frac{C_{t+1}}{B_t^1}\right) = ln\left(1+\frac{B_t^2}{B_t^1}e^{ln(P_{t+1}^1)}\right)$$
$$ln\left(C_{t+1}\right)-ln\left(B_t^1\right) = f(ln(P_{t+1}^1))$$

where $f(x) = ln(1+\alpha e^x)$ with $\alpha = \frac{B_t^2}{B_t^1}$. Notice that for $r_{t+1,t+2} \approx 0$, $P_{t+1}^1 \approx 1$, so $x = ln(P_{t+1}^1) \approx 0$, and the second order Taylor approximation of $f(x)$ around $x = 0$ is:

$$f(x) = ln(\alpha+1)+x\frac{\alpha}{\alpha+1}+x^2\frac{\alpha}{2(\alpha+1)^2}+o[x^2]$$

But x^2 is negligible for small time intervals. First, notice that $x = ln(P_{t+1}^1) = -r_{t+1,t+2} \sim N(\mu_p, \sigma_p^2)$, so $x^2/\sigma_p^2 \sim \mathcal{X}_{1,\mu_p}$, a non-central Chi-squared with 1 degree of freedom and mean $E[\mathcal{X}_{1,\mu_p}] = 1+\mu_p^2$.

$$E[x^2] = \sigma_p^2 E[\mathcal{X}_{1,\mu_p}] = \sigma_p^2(1+\mu_p^2) \approx \sigma_p^2$$
$$Var[x^2] = \sigma_p^4 Var[\mathcal{X}_{1,\mu_p}] = \sigma_p^4 2(1+2\mu_p^2) \approx 2\sigma_p^4 \approx 0$$

So the variance of x^2 is negligible, thus x^2 is approximately equal to its mean: $x^2 \approx E[x^2] = \sigma_p^2$.

So we can replace in the approximation $\alpha = \frac{B_t^2}{B_1^t}$ and $x = ln(P_{t+1})$ and $x^2 \approx E[x^2] \approx \sigma_p^2$ getting:

$$f(ln(P_{t+1}^1)) \approx ln\left(\frac{B_t^2}{B_t^1}+1\right)+ln(P_{t+1}^1)\frac{\frac{B_t^2}{B_t^1}}{\frac{B_t^2}{B_t^1}+1}+\sigma_p^2\frac{\frac{B_t^2}{B_t^1}}{2\left(\frac{B_t^2}{B_t^1}+1\right)^2}$$

Simplifying with some algebra:

$$f(ln(P_{t+1}^1)) \approx ln\left(B_t^2+B_t^1\right)-ln\left(B_t^1\right)+ln(P_{t+1}^1)\frac{B_t^2}{B_t^2+B_t^1}+\sigma_p^2\frac{B_t^2B_t^1}{2\left(B_t^2+B_t^1\right)^2}$$

Hence:

$$ln\left(C_{t+1}\right) \approx ln\left(B_t^2+B_t^1\right)+ln(P_{t+1}^1)\frac{B_t^2}{B_t^2+B_t^1}+\sigma_p^2\frac{B_t^2B_t^1}{2\left(B_t^2+B_t^1\right)^2}$$

Rewriting $b_t \equiv B_t^2/(B_1^t + B_t^2)$:

$$ln\left(C_{t+1}\right) \approx ln\left(B_t^2 + B_t^1\right) + ln(P_{t+1}^1)b_t + 0.5\sigma_p^2 b_t(1-b_t)$$

hence, given that $ln(P_{t+1}^1) \sim N(\mu_p, \sigma_p^2)$, we have:

$$\begin{aligned}
&ln\left(C_{t+1}\right) \approx N(\mu_c, \sigma_c^2) \\
&\qquad \mu_c \equiv ln\left(B_t^2 + B_t^1\right) + \mu_p b_t + 0.5\sigma_p^2 b_t(1-b_t) \\
&\qquad \sigma_p^2 \equiv b_t^2 \sigma_p^2
\end{aligned}$$

Moreover, following Campbell at p.28, we can show that this approximation tends to exact if we take time intervals Δ going to 0. The squared normalized log return scaled by period duration is $x^2/(\Delta\sigma_p^2)$ also has non-central Chi squared distribution and:

$$\begin{aligned}
E[x^2] &= \Delta\sigma_p^2 E[\mathcal{X}_{1,\mu_p}] = \Delta\sigma_p^2(1+\mu_p^2) \approx \Delta\sigma_p^2 \\
Var[x^2] &= \Delta^2\sigma_p^4 Var[\mathcal{X}_{1,\mu_p}] = \Delta^2\sigma_p^4 2(1+2\mu_p^2) \approx 2\Delta^2\sigma_p^4
\end{aligned}$$

As $\Delta \to 0$, the variance of x^2 vanishes compared to the mean of x^2, so $x^2 \to E[x^2] = \sigma_p^2$.

D.2 Proof of Lemma 2:

Using Lemma 1 at the second line below

$$\begin{aligned}
ln\left(C_{t+1}^{-\gamma}P_{t+1}^1\right) &= -\gamma ln\left(C_{t+1}\right) + ln\left(P_{t+1}^1\right) \\
&= -\gamma\left[ln\left(B_t^2 + B_t^1\right) + ln(P_{t+1}^1)b_t + 0.5\sigma_p^2 b_t(1-b_t)\right] + ln\left(P_{t+1}^1\right) \\
&= -\gamma ln\left(B_t^2 + B_t^1\right) + ln(P_{t+1}^1)(-\gamma b_t + 1) - \gamma 0.5\sigma_p^2 b_t(1-b_t)
\end{aligned}$$

Then, since it is an affine transformation of a normal rv, it is also normal:

$$ln\left(C_{t+1}^{-\gamma}P_{t+1}^1\right) \sim N\left(-\gamma ln\left(B_t^2 + B_t^1\right) + \mu_p(1-\gamma b_t) - \gamma 0.5\sigma_p^2 b_t(1-b_t), \qquad (1-\gamma b_t)^2\sigma_p^2\right)$$

Therefore, using the mean of a log-normal:

$$\begin{aligned}
E_t\left[C_{t+1}^{-\gamma}P_{t+1}^1\right] &= e^{-\gamma ln\left(B_t^2+B_t^1\right)+\mu_p(1-\gamma b_t)-\gamma 0.5\sigma_p^2 b_t(1-b_t)+0.5(1-\gamma b_t)^2\sigma_p^2} \\
&= e^{-\gamma\mu_c+\mu_p+0.5(1-2\gamma b_t+\gamma^2 b_t^2)\sigma_p^2} \\
&= e^{-\gamma\mu_c+\mu_p+0.5(1-2\gamma b_t)\sigma_p^2+0.5\gamma^2\sigma_c^2}
\end{aligned}$$

D.3 Theorem 1: Verify Conjecture of log-normal distribution of P_{t+1}^1

Suppose that:

A1. The conjecture holds for $t+2$:
the belief at time t is $ln(P_{t+2}^1) \sim N(\tilde{\mu}_p, \tilde{\sigma}_p^2)$

A2. Endowment is log normal:
The belief at time t is $ln(E_{t+1}) \sim N(\mu_E, \sigma_E^2)$

A3. Small absolute value of interest rate:
The belief at time t is such that the distribution of $ln(P_{t+1})$ has mass close to zero and well below $|1|$, in the sense that $E[ln(P_{t+1})]^2 \approx 0$ and $Var[ln(P_{t+1})^2] \approx 0$.

Then the conjecture also approximately holds for time $t+1$. That is, when A1, A2 and A3 hold, the model implies that $ln(P_{t+1}) \overset{approx}{\sim} N(\mu_p, \sigma_p^2)$ with

$$\mu_p = \frac{(AD+1)}{[ADb_t + \gamma^{-1}]}\left[-\frac{AD(b_t-\gamma^{-1})^2\sigma_E^2}{2\left[ADb_t+\gamma^{-1}\right]^2} + \mu_E + ln(D) - ln(AD+1)\right]$$
$$\sigma_p^2 = \sigma_E^2(AD+1)^2\left[ADb_t + \gamma^{-1}\right]^{-2}$$

where:

$$A \equiv (B_t^2 + B_t^1)e^{0.5\sigma_p^2 b_t(1-b_t)}$$
$$D \equiv \beta^{1/\gamma}e^{-\tilde{\mu}_c + 0.5\gamma\tilde{\sigma}_c^2}$$
$$\tilde{\mu}_c \equiv ln(B_{t+1}^1 + B_{t+1}^2) + \tilde{\mu}_p b_{t+1} + 0.5\tilde{\sigma}_p^2 b_{t+1}(1-b_{t+1})$$
$$\tilde{\sigma}_c^2 \equiv \tilde{\sigma}_p^2 b_{t+1}^2$$
$$b_{t+1} \equiv B_{t+2}^2/(B_{t+1}^1 + B_{t+2}^2) \qquad \blacksquare$$

So while our solution is still implicit (notice σ_p^2 is in the definition of A), clearly μ_p and σ_p^2 are determined by the sequence of outstanding bonds $\{B_\tau^1, B_\tau^2\}_{\tau\geq t}$ through and only through their sums and ratios $\{(B_\tau^1 + B_\tau^2), b_\tau\}_{\tau\geq t}$:

$$\mu_p = \mu_p\left((B_t^1 + B_t^2), (B_{t+1}^1 + B_{t+1}^2), b_t, b_{t+1}; \qquad \mu_E, \sigma_E^2, \beta, \gamma, \tilde{\mu}_p; \tilde{\sigma}_p^2\right)$$
$$\sigma_p^2 = \sigma_p^2\left((B_t^1 + B_t^2), (B_{t+1}^1 + B_{t+1}^2), b_t, b_{t+1}; \qquad \sigma_E^2, \beta, \gamma, \tilde{\mu}_p; \tilde{\sigma}_p^2\right)$$

D.4 Proof of Theorem 1

Under A1, the conjecture holds for $t+1$, so the equilibrium P_{t+1}^1 is given by [19] :

$$P_{t+1}^1 = [E_{t+1} - (B_t^1 + P_{t+1}^1 B_t^2)]^\gamma \beta e^{-\gamma\tilde{\mu}_c + 0.5\gamma^2\tilde{\sigma}_c^2}$$

Define for brevity $D^\gamma \equiv \beta e^{-\gamma\tilde{\mu}_c + 0.5\gamma^2\tilde{\sigma}_c^2}$ and using $C_{t+1,old} = (B_t^1 + P_{t+1}^1 B_t^2)$ with a little algebra:

$$(P_{t+1}^1)^{\frac{1}{\gamma}} = [E_{t+1} - C_{t+1}]D$$
$$(P_{t+1}^1)^{\frac{1}{\gamma}} + C_{t+1}D = E_{t+1}D$$
$$1 + C_{t+1}D/(P_{t+1}^1)^{\frac{1}{\gamma}} = E_{t+1}D/(P_{t+1}^1)^{\frac{1}{\gamma}}$$
$$ln\left(1 + De^{ln(C_{t+1}) - \gamma^{-1}ln(P_{t+1}^1)}\right) = ln(E_{t+1}) + ln(D) - \gamma^{-1}ln(P_{t+1}^1)$$

Now, using A3 on the distribution of $x = ln(P_{t+1})$, we can find an approximation for x^2:

$$ln(P_{t+1}^1)^2 \quad \overset{Var[ln(P_{t+1}^1)^2]\approx 0}{\overset{\downarrow}{\approx}} \quad E[ln(P_{t+1}^1)^2] \quad \overset{E[ln(P_{t+1}^1)]^2\approx 0}{\overset{\downarrow}{\approx}} \quad Var[ln(P_{t+1}^1)] \equiv \sigma_p^2$$

So for $x \equiv ln(P_{t+1}^1)$, we have $x^2 \approx E[x^2] \approx Var(x) \equiv \sigma_p^2$. So Lemma 1 goes though and we can use the approximation

$$ln\,(C_{t+1}) \approx ln\left(B_t^2 + B_t^1\right) + ln(P_{t+1}^1)b_t + 0.5\sigma_p^2 b_t(1-b_t)$$

[19]We just use the previously found implicit solution for the unique equilibrium, applied to period $t+1$ rather than t, and define $\tilde{\mu}_c, \tilde{\sigma}_c^2$ as functions of $\tilde{\mu}_p, \tilde{\sigma}_p^2$ just like we defined μ_c, σ_c^2 as functions of μ_p, σ_p^2 in Lemma 1

Using this approximation to replace C_{t+1} out of the exponent:

$$ln\left(1 + De^{ln\left(B_t^2+B_t^1\right)+ln(P_{t+1}^1)(b_t-\gamma^{-1})+0.5\sigma_p^2 b_t(1-b_t)}\right) \approx ln(E_{t+1}) + ln(D) - \gamma^{-1}ln(P_{t+1}^1)$$
$$ln\left(1 + DAe^{ln(P_{t+1}^1)(b_t-\gamma^{-1})}\right) \approx ln(E_{t+1}) + ln(D) - \gamma^{-1}ln(P_{t+1}^1)$$

Where we defined for brevity

$$A \equiv (B_t^2 + B_t^1)e^{0.5\sigma_p^2 b_t(1-b_t)}$$

Now for the LHS we proceed analogously to Lemma 1, taking the second order Taylor approximation of $f(x) = ln(1 + \alpha e^{Cx})$ around $x = 0$, with $x = ln(P_{t+1})$ and and $\alpha = AD$ and $C = b_t - 1/\gamma$

$$f(x) = ln(\alpha + 1) + \frac{\alpha}{\alpha + 1}Cx + \frac{\alpha}{2(\alpha + 1)^2}C^2x^2 + o(x^2)$$

As already shown, A3 on the distribution of $x = ln(P_{t+1})$, justifies the approximation for $x^2 \approx E[x^2] \approx Var(x) \equiv \sigma_p^2$. Replacing this approximation as well as $\alpha = AD$ and $C = b_t - \gamma^{-1}$:

$$f(ln(P_{t+1}^1)) \approx ln(AD + 1) + \frac{AD}{AD + 1}(b_t - \gamma^{-1})ln(P_{t+1}^1) + \frac{AD}{2(AD + 1)^2}(b_t - \gamma^{-1})^2\sigma_p^2$$

Plugging this back to replace the LHS:

$$ln(AD + 1) + \frac{AD}{AD + 1}(b_t - \gamma^{-1})ln(P_{t+1}^1) + \frac{AD}{2(AD + 1)^2}(b_t - \gamma^{-1})^2\sigma_p^2 \approx ln(E_{t+1}) + ln(D) - \gamma^{-1}ln(P_{t+1}^1)$$

Solving the linear equation for $ln(P_{t+1}^1)$:

$$\left[\frac{AD}{AD + 1}(b_t - \gamma^{-1}) + \gamma^{-1}\right] ln(P_{t+1}^1) \approx -\frac{AD}{2(AD + 1)^2}(b_t - \gamma^{-1})^2\sigma_p^2 + ln(E_{t+1}) + ln(D) - ln(AD + 1)$$
$$(AD + 1)^{-1}\left[ADb_t + \gamma^{-1}\right] ln(P_{t+1}^1) \approx -\frac{AD}{2(AD + 1)^2}(b_t - \gamma^{-1})^2\sigma_p^2 + ln(E_{t+1}) + ln(D) - ln(AD + 1)$$
$$ln(P_{t+1}^1) \approx (AD + 1)\left[ADb_t + \gamma^{-1}\right]^{-1}\left[-\frac{AD}{2(AD + 1)^2}(b_t - \gamma^{-1})^2\sigma_p^2 + ln(E_{t+1}) + ln(D) - ln(AD + 1)\right]$$

So $ln(P_{t+1}^1)$ is (approximatively) a linear affine transformation of $ln(E_{t+1})$. This fact combined with $ln(E_{t+1})$ having normal distribution by A2 implies that $ln(P_{t+1}^1)$ has approximately normal distribution:

$$\Rightarrow ln(P_{t+1}^1) \overset{approx}{\sim} N(\mu_p, \sigma_p^2)$$

We can compute μ_p, σ_p^2 taking expected value and variance of both sides:

$$\mu_p \equiv E[ln(P_{t+1}^1)] = (AD + 1)\left[ADb_t + \gamma^{-1}\right]^{-1}\left[-\frac{AD}{2(AD + 1)^2}(b_t - \gamma^{-1})^2\sigma_p^2 + \mu_E + ln(D) - ln(AD + 1)\right]$$
$$\sigma_p^2 \equiv Var[ln(P_{t+1}^1)] = \sigma_E^2(AD + 1)^2\left[ADb_t + \gamma^{-1}\right]^{-2}$$

Using the second to replace σ_p out of the first:

$$\mu_p \equiv E[ln(P_{t+1}^1)] = \frac{(AD + 1)}{[ADb_t + \gamma^{-1}]}\left[-\frac{AD(b_t - \gamma^{-1})^2\sigma_E^2}{2\left[ADb_t + \gamma^{-1}\right]^2} + \mu_E + ln(D) - ln(AD + 1)\right]$$